

## Analytical Model for Critical Impact Energy of Spalling and Penetration in Concrete Wall

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# Etimaad Engineering (Pvt.) Ltd., Hyderabad, Sindh, Pakistan.

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### Article Info

#### Article history:

Received Apr 2, 2012

Revised May 21, 2012

Accepted June 2, 2012

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#### Keyword:

Concrete  
Critical impact energy  
Penetration  
Projectile  
Nose shape

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### ABSTRACT

Penetration is the basic element of designing protective concrete structure against the local impact of hard projectile. Conventional, un-conventional, and sensitive structures should have to be designed as self-protective structures in order to resist natural disaster, consciously engendered unpleasant incidents, or/and against accidentally occur incidents in nuclear plants, local industries etc.. When hard projectile collides with concrete wall, it is the critical impact energy of the projectile that deforms concrete wall. Critical impact energy is the dominant cause of penetration in concrete structures. Therefore, it is vital to study critical impact energy that causes penetration. An analytical model is developed to predict the required critical impact energy for spalling and tunneling and maximum penetration without rear effects in concrete walls when it is impacted with hard projectile. The newly developed analytical model is examined for CRH =2.0, 3.0. It was found that the predicted results from analytical model are in close relation with experimental data with less than (8%) and (17%) error in case of CRH =2.0 and 3.0. Furthermore, Chen and Li nose shape factor is modified as ( $N_i$ ), with introduction of empirical frictional factor ( $N_f$ ). It was found that the predicted results from analytical model with proposed nose shape ( $N_i$ ) are in close relation with experimental data in all cases as compared to predicted results with traditional Li and Chen nose shape ( $N^*$ ). In general, the analytical model generates encouraging prediction which is consistent and follows a general trend of experimental results. Therefore, it is suggested that the proposed analytical model is conservative.

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## 1. INTRODUCTION

Concrete is a common material for constructing protective structures to resist impact and explosive loads both in civil and military applications. Such as power plants, weapon industries, weapons storage places, water retaining structures, highway structures, and nuclear industry design need to be considered to produce more efficient protection against impact by kinetic projectile, generated both accidentally or deliberately or by natural disasters, in various impact and blast scenarios (e.g. failure of a pressurized vessel, failure of a turbine blade or other high speed rotating machines, aircraft crashes, fragments generated by

accidental explosions, etc.). Hard projectile impact can create local damage to the structure around the contact zone and overall dynamic response of the structure. The projectile may be classified as 'Hard' and 'Soft' depending upon deformability of projectile with respect to target's deformation. Deformation of hard projectile is considerably smaller or negligible compared with target's deformation [6, 11-13,19-25]. Almost in all cases hard projectiles are considered as non – deformable or rigid. However, 'Soft' projectile deforms itself considerably well as compared to target's deformation [6,11-13,19-25]. Penetration process is briefly explained below:

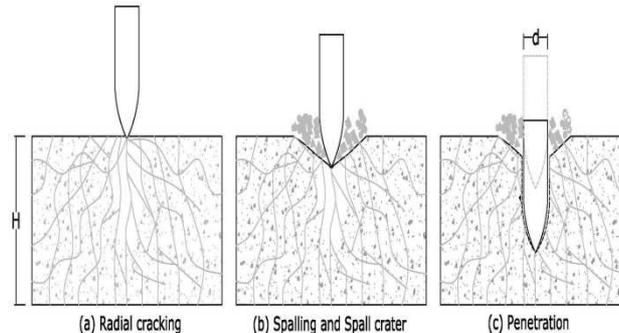


Figure 1. Explains the local impact phenomena caused by hard projectile. (a) Radial cracking, (b) Spalling and Spall crater, (c) Penetration.

**Radial Cracking:** When projectile collides with concrete slab with certain velocity, it results radial cracks originated from the point of impact within the slab in every direction [6,12,19-25].

**Spalling:** The ejection of material of slab from front face (impacted face) due to impact of hard projectile is called spalling. Spalling produces spall crater in the surrounding area of impact. Spall crater is the total damaged portion of peeling off material from slab on impacted face [6,11,12,19-25].

**Tunneling:** The digging of projectile into the concrete wall afar from the thickness of spall crater. The lengthwise measurement of dig is called tunneling depth [12].

**Penetration:** Penetration is complete process including spalling and tunneling of concrete slab. The total penetration depth can be defined as the depth of spall crater and tunneling. [6,11,12,19-25].

Critical impact energy is the dominant cause of penetration in concrete slabs [11,12,19,20]. When a hard projectile impact with concrete slab, the critical impact energy of the projectile is the main reason that makes concrete wall deforms. In general, the critical impact energy of hard projectile penetrating in to concrete slab can be studied in three ways, (i) Empirical Studies (predict empirical formula based on experimental data), (ii) Analytical Studies (create formula based on physical laws), and (iii) Numerical Simulation (based on computer based material model). Analytical modeling approach offers the most economical and most efficient way of predicting the penetration. Once the underpinning of mechanics of penetration of hard projectile on concrete slab understood well, the accurate analytical model can provide almost infinite extent range of validity. In this paper, an analytical model is developed to predict the required critical impact energy for spalling, tunneling, and for maximum penetration without rear effects in concrete walls subjected to impact of hard projectile. Furthermore, a nose shape factor ( $N_i$ ) with introduction of empirical frictional factor ( $N_f$ ) as modification in Chen and Li nose shape ( $N^*$ ) is proposed to determine the effect of nose shape on required critical impact energy which causes the spalling, tunneling, and maximum penetration in to concrete walls against the impact hard projectile.

## 2. LITERATURE REVIEW

The penetration of hard projectile in to concrete structures have been studied since the mid of 17th century because of continuous military interest in designing high performance projectile and high performance protective barriers [6,12,19-25]. A review of previous research work exposed that peak studies on penetration of concrete structures against dynamic loading were conducted from the early 1940s [6,14,19-25]. However, most of the research work ceased shortly after World War – II and were not resumed until 1960s [6,14,19-25]. The intensive study on concrete walls against penetration of hard projectile in the nuclear industry re-initiated since three and half decades ago. Various analytical studies were conducted to specify the penetration of hard projectile on concrete structures, a review of these studies were discussed intensively

in previous publications, Kennedy (1976), Bangash (1993), Williams (1994), Corbett et al. (1996), Q.M Li (2005), Bangash (2009), Ismail *et al.* (2010), and Zaidi *et al.* (2010) [6,12,19-25].

Kennedy (1976), proposed impact force time history theory for the penetration of hard projectile on concrete slabs at velocity ranging from 200 to 1500 ft/sec based on assumption of plastic impact. He suggested formula for influence of slab deformability on hard projectile by using reduced effective projectile calibre density, which is function the ratio of actual projectile weight and effective concrete slab weight [11].

(Forrestal *et al.* 1994), they developed an analytical model for penetration of ogive-nose projectiles in to concrete slabs at normal impact with single dimensionless empirical constant ( $S$ ) depends on unconfined compressive strength ( $f_c$ ) of the concrete. Predictions were in good agreement within the striking velocities between 250 m/s and 800m/s [1].

(Jones and Rule 2000) suggested the optimal nose geometry model for normal impact penetration including the effects of pressure-dependent friction. They found that at low velocities the more friction need sharper nose to achieve maximum depth. However, for higher impact velocities, sharpening of nose only produce excessive friction. For modest friction, the optimal nose geometry was very close to that predicted in the frictionless case [16].

(Chen and Li 2002) developed a non-dimensional formula based on the dynamic cavity-expansion model to predict deep penetration of a non-deformable projectile with different geometrical characteristics into several mediums subjected to a normal impact. The proposed formula depends on two dimensionless numbers ( $N$ ) and ( $I$ ), and shows good agreement with penetration results on metal, concrete and soil for a range of nose shapes and impact velocities as long as projectile keeps rigidity [17].

(Li and Chen 2003) developed a dimensionless formula of penetration depth for a hard projectile impacting small, medium, and deep concrete slabs based on dimensional analysis. They found that ( $X/d$ ) is more dependent on ( $I$ ) than ( $N$ ), especially when ( $I/N$ ) is small [5].

(Chen *et al.* 2004 and 2008) suggested the penetration of a rigid projectile into concrete and reinforced concrete slab when the impact at normal and obliquity direction. A general model, i.e., initial cratering, and tunnelling were developed for concrete slabs based on dynamic cavity expansion theory. The proposed formulae are consistent with other empirical formulae and correlate well with experimental data [7,8].

(Li *et al.* 2006) developed a model for critical impact energy for scabbing and perforation of concrete slabs when impacted with flat nose hard projectile. The predicted results of model were encouraging when compared with NDRC and UMIST formulae and with experimental results [15].

(Guirgis *et al.* 2009) They suggested a dimensionless semi-analytical formula for the penetration depth of a rigid projectile in a concrete slab based on the volumetric crushing energy density. The results of penetration depth compare with Modified NDRC formula on experimental data of Forrestal et al. (1994 and 2003) [18].

Review of previous analytical work reveals that only limited researchers investigated the spalling, tunneling and penetration phenomena of hard projectile on concrete slabs with the vision of critical impact energy. Therefore, an analytical study has been carried out to explore and further improve the prognostic analytical models for spalling, tunneling and penetration of hard projectile on concrete slabs footed on critical impact energy.

### 3. PROPOSED ANAYTICAL MODEL

Consider a rigid projectile having mass ( $m$ ), diameter ( $d$ ) with general nose shape (Fig. 2), impacting on a concrete slab having thickness ( $H$ ) at initial velocity of ( $V_o$ ) at normal direction.

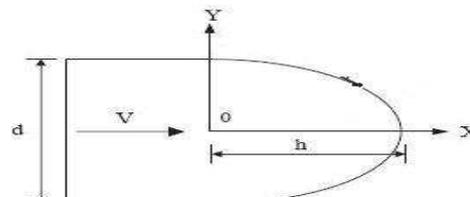


Figure 2. General nose shape.

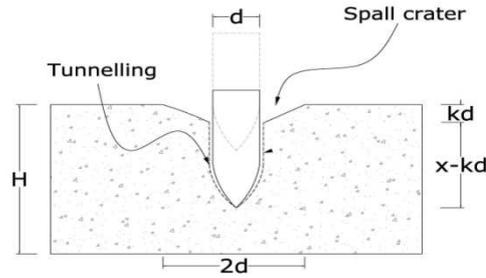


Figure 3. Spalling crater and tunnelling region (penetration) of concrete slab.

As projectile collides with concrete slab, an initial cavity conical crater region known as spall crater formed due to the surface effect. The spall crater is approximately assumed as equal to the 2 times of diameter of projectile in size [1,2,20]. Using dynamic cavity expansion theory spall crater is assumed as a cone with axial depth of  $(kd)$ . Where  $(k)$  is dimensionless parameter and  $(d)$  is diameter of projectile. Forrestal *et al.* in [2,3], and in [4] suggested  $(k = 2.0)$ . However later on, (Li and Chen in 2002 and 2003) suggested that  $k = (0.707 + h/d)$  based on slip-line field theory, in which  $(h)$  is the nose length of projectile as shown in (fig. 2) [5]. The tunnelling region starts from the end of spall crater and continues up to final penetration.

The dynamic cavity expansion theory used to calculate normal stress and axial resistance force on projectile nose. Since the deceleration of projectile is considered as linear, when the interface friction between the projectile nose and concrete slab is neglected, the axial resistance force on the nose of projectile during spalling is [1]:

$$F_R = cx \quad \text{for } \frac{x}{d} < k \quad (1)$$

and during tunnelling:

$$F_R = \frac{\pi d^2}{4} (Sf_c + N^* \rho V^2) \quad \text{for } \frac{x}{d} \geq k \quad (2)$$

Where  $(c)$  is constant, and according to Li and Chen [5]:

$$c = \frac{\pi d^2}{4k} \frac{(Sf_c + N^* \rho V_o^2)}{\left(1 + N^* \rho \left(\frac{\pi k d^3}{4M}\right)\right)} \quad (3)$$

by (Chen *et al.*, 2004 and 2008) in terms of  $(N)$  and  $(I)$  [7,8]:

$$c = \frac{\pi d^2 Sf_c}{4k} \frac{\left(1 + \frac{I}{N}\right)}{\left(1 + \frac{\pi k}{4N}\right)} \quad (4)$$

where

$$I = \frac{MV_o^2}{Sf_c d^3} \quad (5)$$

$$N = \frac{M}{N^* \rho d^3} \quad (6)$$

$$S = 826 f_c^{-0.544} \quad (7)$$

or

$$S = 72 f_c^{-0.5} \quad (8)$$

Where ( $f_c$ ) is in MPa. When ( $N \gg I$ ) and ( $N \gg I$ ), it is further deduced as [8]:

$$c = \frac{\pi d^2 S f_c}{4k} \quad (9)$$

The final penetration depth is given by (Li *et al.* 2005) [12]:

$$x = V_o \sqrt{\frac{M}{c}} \quad \text{for} \left( \frac{x}{d} < k \right) \quad (10)$$

$$x = \frac{2M}{\pi d^2 N^* \rho} \ln \left( 1 + \frac{N^* \rho V_1^2}{S f_c} \right) + kd \quad \text{for} \left( \frac{x}{d} \geq k \right) \quad (11)$$

Where

$$V_1 = \sqrt{\frac{M V_o^2 - \left( \frac{\pi k d^3}{4} \right) S f_c}{M + \left( \frac{\pi k d^3}{4} \right) N^* \rho}} \quad (12)$$

(Li and Chen 2003) modified the equation (10) and (11) for penetration depth by dimensional analysis of non dimensional numbers in terms of ( $N$ ) and ( $I$ ) [5]:

$$\frac{x}{d} = \sqrt{\frac{4k}{\pi} I \frac{\left( 1 + k \frac{\pi}{4N} \right)}{\left( 1 + I/N \right)}} \quad \text{for} \quad \frac{x}{d} < k \quad (13)$$

$$\frac{x}{d} = \frac{2}{\pi} N \ln \left( \frac{1 + I/N}{1 + k \frac{\pi}{4N}} \right) + k \quad \text{for} \quad \frac{x}{d} \geq k \quad (14)$$

When ( $k = 2$ ), equation (13) and (14) provides penetration depth same as in equation (10) and (11). According to (Li and Chen 2003) in practical cases ( $N$ ) is normally much greater than unity; equation (13) and (14) can be simplified as [5]:

$$\frac{x}{d} = \sqrt{\frac{\left( 4kI/\pi \right)}{\left( 1 + I/N \right)}} \quad \text{for} \quad \frac{x}{d} < k \quad (15)$$

$$\frac{x}{d} = \frac{2}{\pi} N \ln \left( 1 + \frac{I}{N} \right) + \frac{k}{2} \quad \text{for} \quad \frac{x}{d} \geq k \quad (16)$$

When ( $I/N \ll 1$ ), equation (15) and (16) can be further simplified as (Li and Chen 2003) [5]:

$$\frac{x}{d} = \sqrt{\left( 4kI/\pi \right)} \quad \text{for} \quad \frac{x}{d} < k \quad (17)$$

$$\frac{x}{d} = \frac{k}{2} + \frac{2I}{\pi} \quad \text{for} \quad \frac{x}{d} \geq k \quad (18)$$

Therefore, the critical impact energy for spall crater can be calculated by:

$$\frac{E_{csp}}{f_c d^3} = \frac{\pi k S}{8} \left( \frac{1 + I/N}{1 + \pi k/4N} \right) \quad \text{for } \frac{x}{d} \geq k \quad (19)$$

The residual impact energy at the end of spall crater which is main reason of causing tunnelling or in other words, the critical impact energy required for tunnelling is based on equation (12) and (11) respectively:

$$\frac{E_{ct}}{f_c d^3} = \frac{E_{rs}}{f_c d^3} = \frac{S}{8} \left( \frac{4I + \pi k}{1 + \pi k/4N} \right) \quad \text{for } \frac{x}{d} \geq k \quad (20)$$

or

$$\frac{E_{ct}}{f_c d^3} = \frac{E_{rs}}{f_c d^3} = \frac{SN}{2} \left( e^{\frac{\pi}{2dN}(x-kd)} - 1 \right) \quad \text{for } \frac{x}{d} \geq k \quad (21)$$

The critical impact energy required to penetrate a concrete slab can be find by using Newton's second law of motion:

$$mV \frac{dV}{dx} = - \frac{\pi d^2}{4} (Sf_c + N^* \rho V^2) \quad (22)$$

Integrating equation (22) from ( $V_i$ ) to (0) and ( $kd$ ) to ( $x$ ) leads the required critical impact energy to penetrate a concrete slab:

$$\frac{E_{cp}}{f_c d^3} = \frac{S}{2} \left( N \left( e^{\frac{\pi}{2dN}(x-kd)} - 1 \right) + \frac{\pi}{2} e^{\frac{\pi}{2dN}(x-kd)} \right) \quad \text{for } \frac{x}{d} \geq k \quad (23)$$

From equation (13) and (14) the critical impact energy for spall crater and penetration can be calculated by:

$$\frac{E_{csp}}{f_c d^3} = \frac{S}{2 \left[ \frac{4kd^2 \left( 1 + \frac{k\pi}{4N} \right)}{x^2 \pi} - \frac{1}{N} \right]} \quad \text{for } \frac{x}{d} < k \quad (24)$$

$$\frac{E_{cp}}{f_c d^3} = \frac{NS}{2} \left[ \left( 1 + \frac{k\pi}{4N} \right) e^{\frac{\pi(x-kd)}{2Nd}} - 1 \right] \quad \text{for } \frac{x}{d} \geq k \quad (25)$$

In practical cases when ( $N$ ) is normally much greater than unity; the critical impact energy for spall crater and penetration equation (24) and (25) can be simplified as:

$$\frac{E_{csp}}{f_c d^3} = \frac{S}{2 \left[ \frac{4kd^2}{x^2 \pi} - \frac{1}{N} \right]} \quad \text{for } \frac{x}{d} < k \quad (26)$$

$$\frac{E_{cp}}{f_c d^3} = \frac{NS}{2} \left[ e^{\frac{\pi}{2N} \left( \frac{x}{d} - \frac{k}{2} \right)} - 1 \right] \quad \text{for } \frac{x}{d} \geq k \quad (27)$$

When ( $I/N \ll 1$ ), the critical impact energy for spall crater and penetration equation (26) and (27) can be

further simplified as:

$$\frac{E_{csp}}{f_c d^3} = \frac{Sx^2 \pi}{8kd^2} \quad \text{for } \frac{x}{d} < k \tag{28}$$

$$\frac{E_{cp}}{f_c d^3} = \frac{S\pi}{4} \left( \frac{x}{d} - \frac{k}{2} \right) \quad \text{for } \frac{x}{d} \geq k \tag{29}$$

**3.1. Proposed Empirical Nose Shape Factor ( $N_i$ ) and ( $N_f$ )**

The effect of nose shape of rigid projectile on local impact effects especially in penetration is important and has been considered in many of the empirical and analytical formulae. However, the definitions of the nose shape factor in each formula are different and without frictional factor.

Nose shape parameter	Value	Definition
<sup>a</sup> $N_{NDRC}$	0.72	Flat
	0.84	Blunt
	1.0	Spherical
	1.14	Sharp nose
<sup>b</sup> $N_{Hughes}$	1.0	Flat
	1.12	Blunt
	1.26	Spherical
	1.39	Sharp nose
<sup>c</sup> $N_{Sandia}$	$0.56 + 0.183\Psi$	Ogive ( $\Psi = R/d$ is the caliber-radius-head (CRH))
	$0.56 + 0.25\Psi$	Conical ( $\Psi = h/d$ where $h$ is the length of the nose head)
<sup>d</sup> $N_{ACE}$	0.56	Flat (nose caliber=0) and the nose caliber is defined by $\sqrt{\Psi - 1/4}$
	0.65	Hemisphere (nose caliber=0.5); $\Psi = r/d$ where $r$ is the radius of the sphere
	0.82	Cone (nose caliber=1)
	0.82	Tangent ogive (nose caliber=1.4)
	0.92	Tangent ogive (nose caliber=2)
	1.0	Tangent ogive (nose caliber=2.4)
	1.08	Cone (nose caliber=2)
	1.11	Tangent ogive (nose caliber=3)
	1.19	Tangent ogive (nose caliber=1.5)
	1.33	Cone (nose caliber=3)
<sup>e</sup> $N^*$	Eq. (2)	Any nose shape [20]
	$\frac{1}{3\Psi} - \frac{1}{24\Psi^2}$	Ogive nose ( $0 < N^* < 0.5$ ) where $\Psi$ is the CRH
	$\frac{1}{1 + 4\Psi^2}$	Conical nose ( $0 < N^* < 1.0$ ) where $\Psi = h/d$ and $h$ is the nose length
	$1 - \frac{1}{8\Psi^2}$	Blunt/spherical nose ( $0.5 < N^* < 1.0$ ) where $\Psi = r/d$ and $r$ is the radius of the sphere

Figure 4. Nose shape factors and formulae.

There are number of frictional forms may can be analyzed during local impact effects of hard projectile. In this research we assume the friction, proportional to the normal pressure as an empirical frictional factor by modifying the Li and Chen [5] ogive nose shape formula. The ogive nose shape formula [5]:

$$N^* = \frac{1}{3\Psi} - \frac{1}{24\Psi^2} \tag{30}$$

Where ( $\Psi$ ) is the CRH ratio, which is equal to ( $S/d$ ). ( $S$ ) is ogive radius and ( $d$ ) is shank diameter of projectile. Therefore, we assume that nose shape factor is equal to:

$$N_i = N^* + N_f \quad (31)$$

Where ( $N^*$ ) for ogive nose projectile from eq. (30). And the ( $N_f$ ) frictional factor of ogive nose projectile is assumed as:

$$N_f = \frac{1}{1.85 \psi^3} \quad \text{for } \psi = 2.0 \quad (32)$$

$$N_f = \frac{1}{1.27 \psi^3} \quad \text{for } \psi = 3.0 \quad (33)$$

Therefore, ( $N_i$ ) for ogive nose rigid projectile is equal to:

$$N_i = \left( \frac{1}{3\psi} - \frac{1}{24\psi^2} \right) + \frac{1}{1.85\psi^3} \quad \text{for } \psi = 2.0 \quad (34)$$

$$N_i = \left( \frac{1}{3\psi} - \frac{1}{24\psi^2} \right) + \frac{1}{1.27\psi^3} \quad \text{for } \psi = 3.0 \quad (35)$$

Therefore eq. (34), and (35) can be used as nose shape factor for required critical impact energy on maximum penetration of a concrete slab without rear effects subjected to the impact of hard projectile.

#### 4. RESULTS AND ANALYSIS

The study on required critical impact energy for maximum penetration of concrete slab is conducted for thick concrete slabs ( $x/d > k$ ) without generating rear effects, on the experimental data of Forrestal et al. ([1], [3], [4], [9], [10]). It is shown that the proposed analytical models are consistently predict the closer bound of experimental results and produced a similar general trend of experimental results in the whole range of experiments presented in Figure (5, and 6).

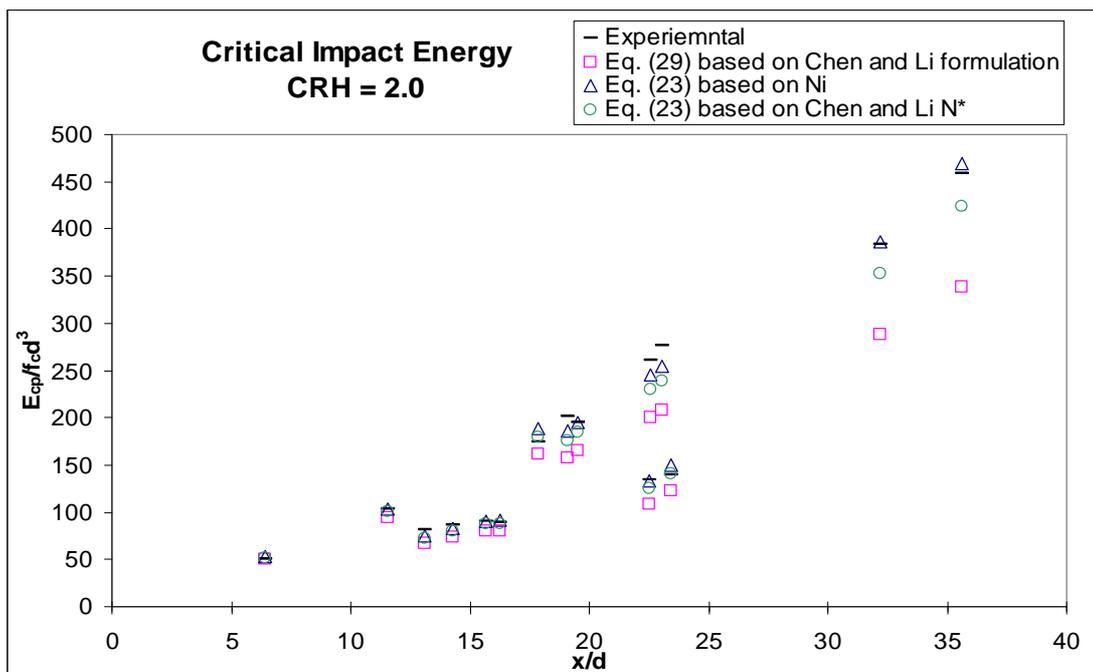


Figure 5. Comparison of results of Eq. (23) by using  $N^*$  and  $N_i$ , and Eq. (29) predictions, and experimental data of the critical impact energy of ogive nose projectile having CRH=2.0 for penetration.

In case of CRH ratio = 2.0, the analytical model eq. (23) with nose shape factor ( $N_i$ ) is able to generate more accurate results with error less than (8%). On the other hand, the analytical model eq. (23) with traditional Chen and Li ( $N^*$ ) generates lower bound less accurate results with error less than (14%). The analytical model based on Chen and Li [5] formulation eq. (29) generates lower bound results with maximum error of (27%) at high velocities.

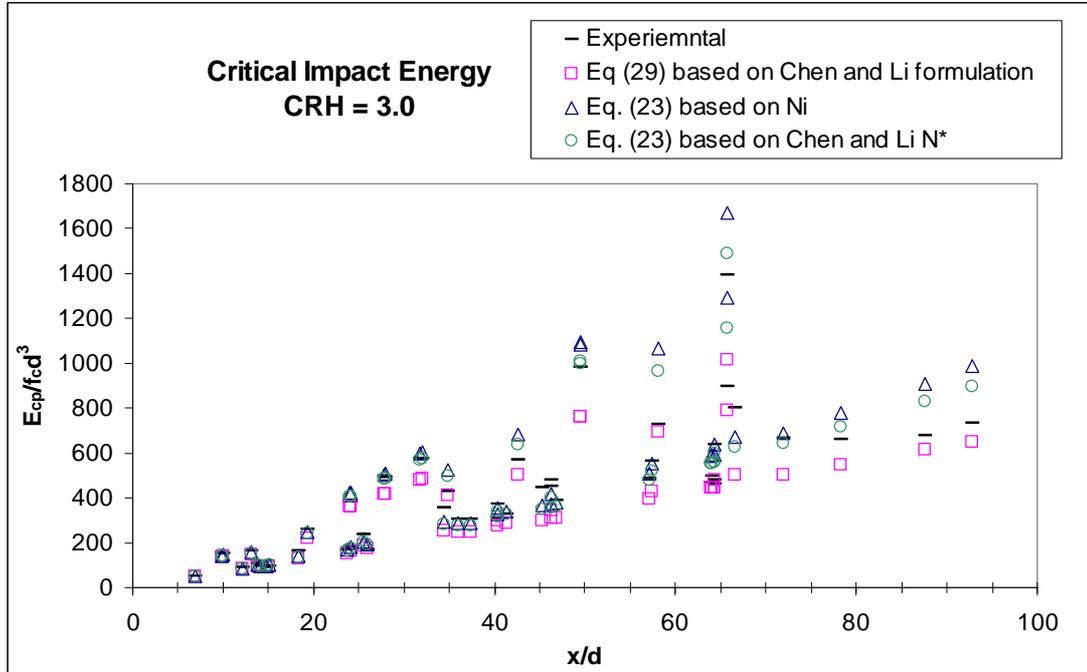


Figure 6. Comparison of results of Eq. (23) by using  $N^*$  and  $N_i$ , and Eq. (29) predictions, and experimental data of the critical impact energy of ogive nose projectile having CRH=3.0 for penetration.

In case of CRH ratio = 3.0, the analytical model eq. (23) with nose shape factor ( $N_i$ ) generates more accurate results with error less than (17%). While the analytical model eq. (23) with traditional Chen and Li ( $N^*$ ) generates results with error less than (22%), and the analytical model based on Chen and Li formulation eq. (29) generates lower bound results with error of (37%) at high velocities.

## 5. CONCLUSION

The analytical model is developed to predict the required critical impact energy to penetrate concrete slab impacted with ogive nose hard projectile. Nose shape factor ( $N_i$ ) has been introduced for CRH = 2.0, and 3.0, and compared with traditional Li and Chen nose shape factor ( $N^*$ ) to know the influence of nose shape factor over the critical impact energies, by comparing analytical predictions with experimental data. The newly developed analytical model is examined for CRH = 2.0, and 3.0. It was found that the predicted results from analytical model are in close relation with experimental data with less than (8%) and (17%) error in case of CRH = 2.0, and 3.0. Furthermore, a nose shape factor ( $N_i$ ) with introduction of empirical frictional factor ( $N_f$ ) as modification in Chen and Li nose shape ( $N^*$ ) is proposed. It was found that the predicted results from analytical model with nose shape ( $N_i$ ) are in close relation with experimental data in all cases as compared to predicted results with traditional Li and Chen nose shape ( $N^*$ ). Generally, the analytical model generates encouraging prediction which is consistent and follows a general trend of experimental results. It is observed in all cases that the error in results of eq. (29) increases with increase in velocity. Therefore, it is suggested that the proposed analytical model eq. (23) is conservative.

## NOMENCLATURE

$d$	(cylindrical) projectile shank diameter.
$E_{ct}$	Critical impact energy of projectile for tunnelling process.
$E_{csp}$	Critical impact energy of the projectile for spalling.
$E_{cp}$	Critical impact energy of the projectile for penetration.
$E_{rs}$	Residual impact energy of the projectile for spalling.
$M$	Mass of the projectile.

$V_o$	Impact velocity of projectile.
$x$	Penetration Depth.
$\rho$	Density of concrete wall.
$H$	Nose shape length of projectile.
$F_r$	Axial resistance force on projectile nose.
$N^*$	Chen and Li Nose shape factor
$V_I$	Impact velocity of projectile for tunnelling.
$N_f$	Empirical nose shape frictional factor.
$N_i$	Nose shape factor (Ismail and Imran)
$\psi$	CRH ratio (S/d).
$N$	Geometry function.
$I$	Impact function.
$H$	Thickness of concrete slab.

### ACKNOWLEDGEMENTS

The authors would like to thank University Tun Hussein Onn Malaysia (UTHM) for supporting this research through International Students Scholarships.

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