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SU(3) Symmetry in hafnium isotopes with even neutron N=100-108

I. Hossain[1], Fadhil I. Sharrad[2], Huda H. Kassim[3], Amir A. Mohammed-Ali[4], A. S. Ahmed[5]
[1,5]Department of Physics, Rabigh College of Science & Arts, King Abdulaziz University, Saudi Arabia
[2,3,4]Department of Physics, College of Science, Kerbala University, Iraq
[2]College of Health and Medical Technology, Al-Ayen University, Iraq

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ABSTRACT

In this paper, we have reviewed the calculation of ground states energy level up to spin 14+, electric quadrupole moments up to spin 12+, and reduced transition probabilities of Hafnium isotopes with even neutron N = 100-108 by Interacting Boson Model (IBM-1). The calculated results are compared with previous available experimental data and found good agreement for all nuclei. Moreover, we have studied potential energy surface of those nuclei. The systematic studies of quadrupole moments, reduced transition strength, yrast level and potential energy surface of those nuclei show an important property that they are deformed and have dynamical symmetry SU(3) characters.

Corresponding Author:

I. Hossain,
Department of Physics, Rabigh College of Science & Arts,
King Abdulaziz University,
21911 Rabigh, Saudi Arabia.
Email: mihossain@kau.edu.sa

1. INTRODUCTION

The interacting boson model offers a simple Hamiltonian, capable of describing collective nuclear properties across a broad range of nuclei, based on general algebraic group theoretical techniques which have also recently found application in problems in atomic, molecular, and high energy physics [1-4]. The three limiting symmetries of this Hamiltonian, U(5), SU(3), and O(6), correspond to the geometrical shapes, spherical vibrator, symmetric rotor, and γ-unstable rotor, respectively [5]. The even-mass Hafnium $^{172-180}$Hf isotope shave been extensively investigated experimentally using a wide variety of reactions. The excited states in the even-even $^{172-180}$Hf isotopes have been investigated from (γ, γ'), (d, p), (α, 2nγ), (α, 4nγ), (n, γ), (n, n'γ) and Coulomb excitation reactions which gave information about the experimental energy levels and the electromagnetic transition probabilities B(E2) in these isotopes [6-15].

In this study, the calculations of energy levels of even-even $^{172-180}$Hf isotopes have been done by using interacting boson model. The ground state band, the reduce probabilities of *E*2 transitions (*B*(*E*2)values), and electric quadrupole moment Q_L are calculated and compared with available experimental data.

2. THEORETICAL MODEL
2.1 Interacting boson model (IBM)

The IBM-1 Hamiltonian can be expressed as [16, 17]:

$$\mathrm{H} = \varepsilon_s(s^\dagger.\tilde{s}) + \varepsilon_d(d^\dagger.\tilde{d}) + \sum_{L=0,2,4}\tfrac{1}{2}(2L+1)^{\frac{1}{2}} C_L\left[[d^\dagger \times d^\dagger]^{(L)} \times [\tilde{d}\times\tilde{d}]^{(L)}\right]^{(0)} + \tfrac{1}{\sqrt{2}}\upsilon_2\left[[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d}\times\tilde{s}]^{(2)} + [d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d}\times\tilde{d}]^{(2)}\right]^{(0)} + \tfrac{1}{2}\upsilon_0\left[[d^\dagger \times d^\dagger]^{(0)} \times [\tilde{s}\times\tilde{s}]^{(0)} + [s^\dagger \times s^\dagger]^{(0)} \times [\tilde{d}\times\tilde{d}]^{(0)}\right]^{(0)} + \tfrac{1}{2}u_0\left[[s^\dagger \times s^\dagger]^{(0)} \times [\tilde{s}\times\tilde{s}]^{(0)}\right]^{(0)} + u_2\left[[d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d}\times\tilde{s}]^{(2)}\right]^{(0)} \quad (1)$$

Then the IBM-1 Hamiltonian in (1) can be written in general form as [18-20]:

$$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \quad (2)$$

Where $\hat{n}_d = (s^\dagger, d^\dagger)$ is the total number of d_{boson} operator, $\hat{p} = 1/2\left[(\tilde{d}.\tilde{d}) - (\tilde{s}.\tilde{s})\right]$ is the pairing operator, $\hat{L} = \sqrt{10}[d^\dagger \times \tilde{d}]^1$ is the angular momentum operator, $\hat{Q} = [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} + \chi[d^\dagger \times \tilde{d}]^{(2)}$ is the quadrupole operator (χ is the quadrupole structure parameter and take the values 0 and $\pm\frac{\sqrt{7}}{2}$ [19, 20]), $\hat{T}_r = [d^\dagger \times \tilde{d}]^{(r)}$ is the octoupole (r=3) and hexadecapole (r=4) operator, and e = e_d - e_s is the boson energy. The parameters are a_0 the strength of the pairing, a_1 angular momentum, a_2 quadrupole, a_3 octoupole and a_4 hexadecapole interaction between the bosons.

3. RESULTS AND DISCUSSION
3.1. Ground state band

In Figure 1 shows that the even-even $^{174-180}$Hf isotopes have a rotational (deformed nuclei) dynamical symmetry SU(3). The rotational limit of the IBM-1 has been applied for the even-even $^{172-180}$Hf isotopes due to the values of the $E_{4_1^+}/E_{2_1^+}$ ratio($E_{4_1^+}/E_{2_1^+} = 3.33$) [19, 20]. The best fitting for the Hamiltonian parameters are presented in Table 1 which gives the best agreement with the experimental data [21-26]. In the framework of the IBM-1, the isotopic chains of Hafnium (Z = 72) nuclei, having a number of proton-bosons holes 5, a number of neutron-bosons holes are (9, 10, 11) for even $^{172-176}$Hf, and (10, 9) for even $^{178-180}$Hf isotopes, respectively.

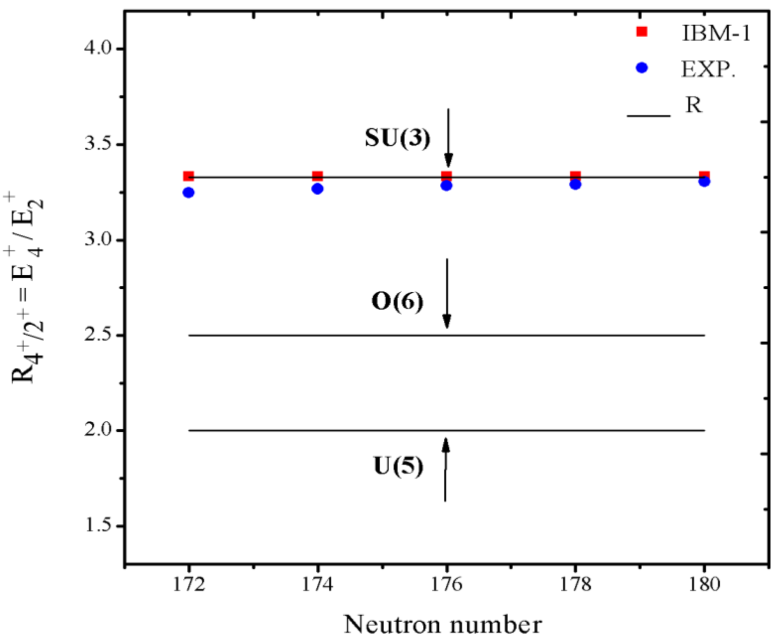

Figure 1. Comparison the IBM-1 calculations with the available experimental data [22-28] of the $E_{4_1^+}/E_{2_1^+}$ ratio for even-even $^{172-180}$Hf nuclei.

Table 1.The parameters of even-even $^{172-180}$Hf nuclei.

Nuclei	*N*	e MeV	a_0 MeV	a_1 MeV	a_2 MeV	a_3 MeV	a_4 MeV	*CHQ* (χ)
^{172}Hf	14	0.00	0.00	0.024	-0.021	0.00	0.00	-1.33
^{174}Hf	15	0.00	0.000	0.023	-0.019	0.00	0.00	-1.33
^{176}Hf	16	0.00	0.000	0.020	-0.0242	0.00	0.00	-1.33
^{178}Hf	15	0.00	0.000	0.021	-0.0258	0.00	0.00	-1.33
^{180}Hf	14	0.00	0.000	0.021	-0.0268	0.00	0.00	-1.33

Figure 2 indicates the energy of ground state band in experimental and theoretical data. This figure has shown the IBM-1 calculations for ground band (energies, spin and parity) in good agreement with those of the experimental data [22-28].

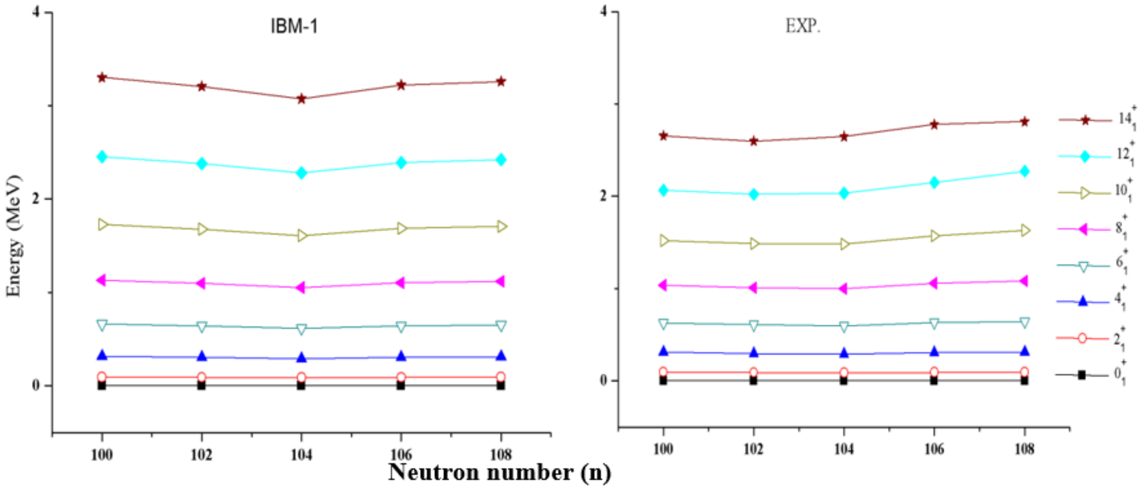

Figure 2. Ground state energy level in the IBM-1 calculations and experimental data [22-28] of even-even $^{172-180}$Hf isotopes.

3.2. B(E2) and Q_L value

The general form of the electromagnetic transitions operator in IBM-1is [19, 20, 29]:

$$\hat{T}^{(L)} = \gamma_0[\hat{s}^+ \times \hat{s}^\sim]^{(0)} + \alpha_2[\hat{d}^+ \times \hat{s}^\sim + \hat{s}^+ \times \hat{d}^\sim]^{(2)} + \beta_L[\hat{d}^+ \times \hat{d}^\sim]^{(L)} \quad (3)$$

Where γ_0, α_2 and β_L (L = 0, 1, 2, 3, 4) are parameters specifying the various terms in the corresponding operators. Equation (4) yields transition operators for E2 transitions with appropriate values of the corresponding parameters.[19]:

$$T^{E2} = \alpha_2 [d^\dagger s + s^\dagger d]^{(2)} + \beta_2[d^\dagger d]^{(2)} \quad (4)$$

Where ($s^\dagger$, $d^\dagger$) and (s, d) are creation and annihilation operators for s and d bosons, respectively [30], while $\alpha_2$and $\beta_2$are two adjustable parameters that measure the strength of each term. The electric transition probabilities, B(E2) values are defined in terms of reduced matrix elements by Iachello and Arima (1987) as [20,29]:

$$B(E2, J_i \rightarrow J_f) = \frac{1}{2J_i+1}\left|\langle J_f \| T^{E2} \| J_i \rangle\right|^2 \quad (5)$$

For the calculations of the absolute B(E2) values, the parameters, α_2 and β_2 of (4), were adjusted according to reproduce the experimental B(E2;$2_1^+ \rightarrow 0_1^+$). Table 2 shows the values of the α_2 and β_2 parameters, which were obtained in the present calculations. The comparison of calculations values of B(E2) transitions with experimental data[22-29], are given in Table 3, for all isotopes under study.

Table 2. Effective charge (in eb) used to reproduce B(E2)valuesfor $^{172\text{-}180}$Hf nuclei.

A	N	α_2	β_2
^{172}Hf	14	0.1004	-0.2969
^{174}Hf	15	0.0940	-0.2780
^{176}Hf	16	0.0980	-0.2898
^{178}Hf	15	0.0980	-0.2898
^{180}Hf	14	0.1038	-0.3071

Table 3.The B(E2) values for $^{172\text{-}180}$Hf nuclei (in e^{2}b^2).

$J_i \to J_f$	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
	^{172}Hf		^{174}Hf		^{176}Hf		^{178}Hf		^{180}Hf	
$2_1^+ \to 0_1^+$	0.8748	0.8754	0.8735	0.8775	1.0689	1.0726	0.9493	0.9521	0.9353	0.9361
$2_2^+ \to 0_1^+$	0.0000	--	0.0000	0.0013	0.0000	0.0005	0.0058	0.0043	0.0073	--
$2_2^+ \to 0_2^+$	0.6845	--	0.0053	--	0.0059	--	1.3424	--	1.3208	1.3891
$2_3^+ \to 0_2^+$	0.0270	--	0.7092	--	0.8647	--	0.4562	--	0.4389	--
$4_1^+ \to 2_1^+$	1.2353	--	1.2351	--	1.5129	--	1.4511	1.3746	1.4242	1.4556
$4_1^+ \to 2_2^+$	0.0000	--	0.0000	0.0416	0.0000	0.0140	0.8797	--	0.8279	--
$4_2^+ \to 2_2^+$	0.0294	--	0.4197	--	0.5198	--	1.4776	1.4460	1.4447	1.5099
$6_1^+ \to 4_1^+$	1.3320	--	1.3352	--	1.6384	--	1.0258	--	0.8430	--
$6_2^+ \to 4_2^+$	0.7624	--	0.8094	--	1.0047	--	1.4620	1.4817	1.4219	1.4495
$8_1^+ \to 6_1^+$	1.3512	--	1.3596	--	1.6728	--	1.0652	--	0.9466	--
$8_2^+ \to 6_2^+$	0.9183	--	0.9438	--	1.1756	--	1.4178	1.7140	1.3692	1.3408
$10_1^+ \to 8_1^+$	1.3299	--	1.3452	--	1.6611	--	1.0499	--	0.9141	--
$10_2^+ \to 8_2^+$	0.0278	--	0.9801	--	1.2267	--	1.3593	--	1.3397	--
$12_1^+ \to 10_1^+$	1.2806	--	1.3045	--	1.6185	--	1.2356	--	1.2179	--
$12_2^+ \to 10_3^+$	0.9011	--	0.0021	--	0.0022	--	1.2106	--	1.1935	--
$2_1^+ \to 2_1^+$	1.2529	--	1.2506	--	1.5299	--	1.2012	--	1.1846	--
$4_1^+ \to 4_1^+$	1.1390	--	1.1368	--	1.3904	--	1.1964	--	1.1803	--
$6_1^+ \to 6_1^+$	1.1162	--	1.1139	--	1.3618	--	1.1933	--	1.1779	--
$8_1^+ \to 8_1^+$	1.1078	--	1.1053	--	1.3504	--	0.9493	0.9521	0.9353	0.9361
$10_1^+ \to 10_1^+$	1.1038	--	1.1008	--	1.3438	--	0.0058	0.0043	0.0073	--
$12_1^+ \to 12_1^+$	1.1015	--	1.0980	--	1.3386	--	1.3424	--	1.3208	1.3891

Table 3 shows that, in general, most of the calculated results in IBM-1 reasonably consistent with the available experimental data, except for few cases that deviate from the experimental data.
The quadrupole moment (Q_L) is an important property for nuclei that can determine if the nucleus is spherical (Q=0), deformed oblate (Q < 0) or prolate (Q > 0) shapes. The electric quadrupole moments of the nuclei can be derived from the transition rate B(E2,$L_i \to L_f$) values according to (6) [31]:

$$Q_L = [16\pi/5]^{1/2}[L(2L-1)/(2L+1)(L+1)(2L+3)]^{1/2}[B(E2, L_i \to L_f)]^{1/2} \tag{6}$$

Where L is the angular momentum. Table 4 presents the calculation of the electric quadrupole moment Q_L within the framework of IBM-1 for the even-even Hf nuclei. The presented results for Q_L are compared with previous experimental results [32].

Table 4. The electric quadrupole moment Q_L(in eb)

Q_L	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.	IBM-1	EXP.
	^{172}Hf		^{174}Hf		^{176}Hf		^{178}Hf		^{180}Hf	
2_1^+	-1.8971	--	-1.8953	--	-2.0963	-2.10(2)	-1.9759	-2.02(2)	-1.9616	-2.00(2)
2_2^+	1.7001	--	1.7141	-	1.9059	-	1.7870	-	1.7639	--
4_1^+	-2.4144	--	-2.4121	-	-2.6676	-	-2.5147	-	-2.4966	--
6_1^+	-2.6559	--	-2.6531	-	-2.9335	-	-2.7659	-	-2.7463	-
8_1^+	-2.7956	--	-2.7924	-	-3.0865	-	-2.9111	-	-2.8908	-
10_1^+	-2.8868	--	-2.8829	-	-3.1851	-	-3.0054	-	-2.9851	-
12_1^+	-2.9509	--	-2.9461	-	-3.2530	-	-3.0713	-	-3.0514	-

3.3. Potential energy surface (PES)

In recent years, the potential energy surface by Skyrme mean field method was mapped onto the PES of the IBM Hamiltonian[33-36].The expectation value of the IBM-1 Hamiltonian with the coherent state ($|N,\beta,\gamma\rangle$)is used to create the IBM energy surface [20, 37]. The state $|N,\beta,\gamma\rangle$is a product of boson creation operators ($b_c^\dagger$) over the boson vacuum$|0\rangle$, i.e.

$$|N,\beta,\gamma\rangle = 1/\sqrt{N!}\left(b_c^\dagger\right)^N|0\rangle \tag{7}$$

With

$$b_c^\dagger = (1+\beta^2)^{-1/2}\left\{s^\dagger + \beta\left[\cos\gamma\left(d_0^\dagger\right) + \sqrt{1/2}\sin\gamma\left(d_2^\dagger + d_{-2}^\dagger\right)\right]\right\} \tag{8}$$

The energy surface, as a function of βandγ, has been given by [10]

$$E(N,\beta,\gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2}(\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\,\beta^2 + \alpha_4) \tag{9}$$

Where the α_i's are related to the coefficients C_L, v_2, v_0, u_2 and u_0 of (1).β Measures the total deformation of nucleus, when $\beta = 0$, the shape is spherical, and when $\beta \neq 0$ the shape is distorted. γ is the amount of deviation from the focus symmetry and correlates with the nucleus. If γ = 0 the shape is prolate, else if γ = 60 the shape becomes oblate.

The calculated potential energy surfaces for the even-even $^{172\text{-}180}$Hf are presented in Figure 3.From this figure all nuclei are deformed and have rotational-like characters. The prolate deformation is deeper than oblate in all nuclei.

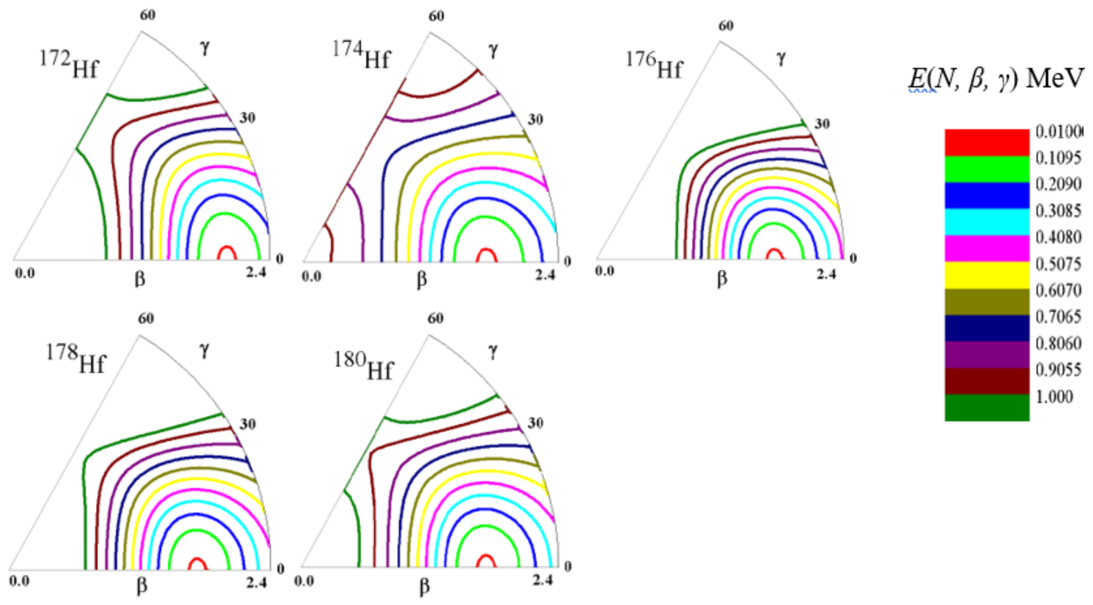

Figure 3. The potential energy surfaces for even-even $^{172\text{-}180}$Hf nuclei.

4. CONCLUSION

We have reviewed theoretical calculations of $^{172\text{-}180}$Hf isotopes with N= 100, 102, 104, 106 and 108 using IBM-1. The even-even $^{176\text{-}180}$Hf isotopes have bosons total numbers of 14, 15, 16, 15 and 14. They were considered fully rotational (fully deformed) nuclei, and the dynamical symmetry of these isotopes is SU(3). The low-lying ground states, electric transition probabilities B(E2), and electric quadrupole moment Q_L are obtained for these isotopes using IBM-1 were compared with the available experimentally data. A good agreement was obtained between theoretical IBM-1for all the observable studied. The potential energy surfaces for Hf isotopes shows that all nuclei are deformed and have dynamical symmetry SU(3) characters.

APPENDIX

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