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A nulling-resistor output amplifier in the framework of heterostructures based on nonlinear partial differential equations

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ABSTRACT

In this paper, we introduce an approach to increase the density of field-effect transistors framework nulling-resistor output amplifier. In the framework of the approach, we consider manufacturing the amplifier in a heterostructure with a specific configuration. Several required areas of the heterostructure should be doped by diffusion or ion implantation. After that dopant and radiation defects should be annealed framework optimized scheme. We also consider an approach to decrease the value of mismatch-induced stress in the considered heterostructure. We introduce an analytical approach to analyze mass and heat transport in heterostructures during the manufacturing of integrated circuits with account for mismatch-induced stress.

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1. INTRODUCTION

At present several actual problems of the solid-state electronics (such as increasing performance, reliability, and density of elements of integrated circuits: diodes, field-effect, and bipolar transistors) are intensively solving [1]–[6]. To increase the performance of these devices, it is attracted an interest determination of materials with higher values of charge carrier's mobility [7]–[10]. One way to decrease the dimensions of elements of integrated circuits is by manufacturing them in thin-film heterostructures [3]–[6]. In this case, it is possible to use inhomogeneity of heterostructure and necessary optimization of doping of electronic materials [11] and development of epitaxial technology to improve these materials (including analysis of mismatch induced stress) [12]–[14]. An alternative approach to increasing the dimensions of integrated circuits is the use of laser or microwave types of annealing [15]–[18].

The main objective of the present paper is to formulate a condition to decrease the dimensions of elements (first of all field-effect heterotransistors) of a nulling-resistor output amplifier as shown in Figure 1, Figure 1(a) shows the structure of the considered nulling-resistor output amplifier and Figure 1(b) shows heterostructure with a substrate, epitaxial layers and buffer layer (view from the side), manufactured in the framework of a heterostructure with a spatial configuration: we consider a heterostructure, which consists of a substrate and an epitaxial layer; the epitaxial layer includes into itself several sections, which were manufactured by using other materials; the obtained sections were doped by diffusion or by ion implantation to obtain the required type of conductivity (p or n) and used as sources, drains and gates of field-effect transistors. The accompanying aim of the present paper is an analysis of the possibility to increase the density of these transistors. Heterostructures have mismatch-induced stress due to the difference between values of lattice distances of the substrate and the epitaxial layer. The stress could lead to dislocation inconsistencies in the substrate and epitaxial layer. In the framework of the paper, we formulate conditions to decrease mismatch-induced stress. The decrease is another accompanying aim of the present paper. To make a prognosis of manufacturing of the considered amplifier we introduce an analytical approach for the prognosis of mass and heat transport during technological processes without crosslinking of solutions on interfaces between layers in the considered heterostructure. The introduced analytical approach gives a possibility to take into account the dependence of parameters of technological parameters on spatial coordinates as well as the dependence of these parameters on time and at the same time nonlinearity of considered processes.

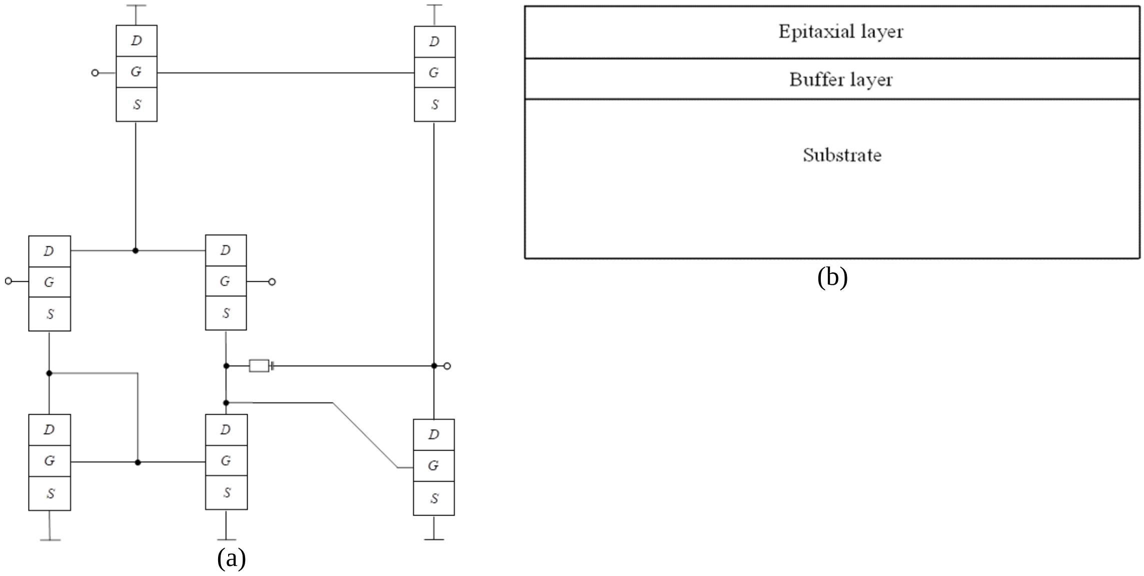

Figure 1. Formulation of condition to decrease dimensions of the elements of (a) Structure of the considered nulling-resistor output amplifier and (b) heterostructure with a substrate, epitaxial layers, and buffer layer (view from the side)

2. METHOD OF SOLUTION

To solve the main aim of the present paper we consider a heterostructure, which consists of a substrate and an epitaxial layer as shown in Figure 1. We also consider a buffer layer between the substrate and the epitaxial layer. The epitaxial layer includes into itself several sections, which were manufactured using another material. These sections have been doped by diffusion or ion implantation to manufacture the required types of conductivity (p or n). These areas became sources, drains, and gates as shown in Figure 1. After the doping, is required annealing of dopant and/or radiation defects. We analyzed the redistribution of dopant and radiation defects to determine conditions, which correspond to decreasing dimensions the considered of the field-effect transistors in the framework of the considered amplifier and at the same time increasing their density. At the same time, we consider the possibility to decrease mismatch-induced stress. To make the analysis we determine and analyze the distribution of concentration of dopant in space and time in the considered heterostructure. We determine the distribution by solving the second Fick's law in the following form [1], [19]–[23] with boundary and initial conditions.

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right] \tag{1}$$

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,$$

$$\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0,\ \text{C (x, y, z,0) =fC (x, y, z)}$$

Where $C\ (x,y,z,t)$ is the Spatio-temporal distribution of concentration of dopant; Ω is the atomic volume of dopant; ∇_s is the symbol of surficial gradient; $\int_0^{L_z} C(x,y,z,t)\,dz$ is the surficial concentration of dopant on the interface between layers of heterostructure (in this situation we assume, that Z-axis is perpendicular to the interface between layers of heterostructure); $\mu_1(x,y,z,t)$ is the chemical potential due to the presence of mismatch-induced stress; D and D_S are the coefficients of volumetric and surficial diffusions. Values of dopant diffusions coefficients depend on properties of materials of the heterostructure, speed of heating and cooling of materials during annealing, and Spatio-temporal distribution of concentration of dopant. Dependences of dopant diffusions coefficients on parameters could be approximated by the following relations [21]–[23].

$$D_C=D_L(x,y,z,T)\left[1+\xi\frac{C^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right],$$

$$D_S=D_{SL}(x,y,z,T)\left[1+\xi_S\frac{C^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right] \tag{2}$$

Where $D_L\ (x,y,z,T)$ and $D_{LS}\ (x,y,z,T)$ are the spatial (due to accounting for all layers of heterostructures) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficients; T is the temperature of annealing; $P\ (x,y,z,T)$ is the limit of solubility of dopant; parameter γ depends on properties of materials and could be an integer in the following interval $\gamma \in$[1], [3], [21]; $V\,(x,y,z,t)$ is the Spatio-temporal distribution of concentration of radiation vacancies; V^* is the equilibrium distribution of vacancies. The concentration dependence of the dopant diffusion coefficient has been described in detail in [21]. Spatio-temporal distributions of concentration of point radiation defects have been determined by solving the following system of equations [19], [22], [23] with boundary and initial conditions.

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]-k_{I,I}(x,y,z,T)\times$$

$$\times I^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t) \tag{3}$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]-k_{V,V}(x,y,z,T)\times$$

$$\times V^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]-k_{I,V}(x,y,z,T)\,I(x,y,z,t)\,V(x,y,z,t)$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

I (x,y,z,0)=fI (x,y,z), V (x,y,z,0)=fV (x,y,z) (4)

Where I (x,y,z,t) is the spatio-temporal distribution of concentration of radiation interstitials; I^* is the equilibrium distribution of interstitials; $D_I(x,y,z,T)$, $D_V(x,y,z,T)$, $D_{IS}(x,y,z,T)$, $D_{VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of interstitials and vacancies, respectively; terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation of divacancies and diinterstitials, respectively (see, for example, [23] and appropriate references in this book); $k_{I,V}(x,y,z,T)$, $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ are the parameters of recombination of point radiation defects and generation of their complexes. Spatio-temporal distributions of divacancies $\Phi_V(x,y,z,t)$ and diinterstitials $\Phi_I(x,y,z,t)$ could be determined by solving the following system of equations [19], [22]–[24].

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+ \tag{5}$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)dW\right]+k_{V,V}(x,y,z,T)\,V^2(x,y,z,t)+k_V(x,y,z,T)\,V(x,y,z,t)$$

With boundary and initial conditions,

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$

$$\left.\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

ΦI (x,y,z,0)=fΦI (x,y,z), ΦV (x,y,z,0)=fΦV (x,y,z) (6)

Where $D_{\Phi I}(x,y,z,T)$, $D_{\Phi V}(x,y,z,T)$, $D_{\Phi IS}$ (x,y,z,T) and $D_{\Phi VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of complexes of radiation defects; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of complexes of radiation defects. Chemical potential μ_1 in (1) could be determine by the following relation [19].

$$\mu_1=E(z)\Omega\sigma_{ij}[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)]/2 \quad (7)$$

Where $E(z)$ is *the* Young modulus, σ_{ij} is the stress tensor; $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ is the deformation tensor; u_i, u_j are the components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; x_i, x_j are the coordinate x, y, z. The (3) could be transformed to the following form,

$$\mu(x,y,z,t)=E(z)\frac{\Omega}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\left\{\frac{1}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]-\varepsilon_0\delta_{ij}+\right.$$

$$\left.+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\left[\frac{\partial u_k(x,y,z,t)}{\partial x_k}-3\varepsilon_0\right]-K(z)\beta(z)\left[T(x,y,z,t)-T_0\right]\delta_{ij}\right\}$$

Where σ is the Poisson coefficient; $\varepsilon_0=(a_s-a_{EL})/a_{EL}$ is the mismatch parameter; a_s, a_{EL} are lattice distances of the substrate and the epitaxial layer; K is the modulus of uniform compression; β is the coefficient of thermal expansion; T_r is the equilibrium temperature, which coincides (for our case) with room temperature. Components of displacement vector could be obtained by solution of the following equations [25].

$$\begin{cases}\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\frac{\partial\sigma_{xx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{xy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{xz}(x,y,z,t)}{\partial z}\\ \rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{\partial\sigma_{yx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{yy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{yz}(x,y,z,t)}{\partial z}\\ \rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{\partial\sigma_{zx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{zy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{zz}(x,y,z,t)}{\partial z}\end{cases}$$

Where,

$$\sigma_{ij}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}-\frac{\delta_{ij}}{3}\frac{\partial u_k(x,y,z,t)}{\partial x_k}\right]+K(z)\delta_{ij}\frac{\partial u_k(x,y,z,t)}{\partial x_k}-K(z)\times$$

$\times\beta(z)\left[T(x,y,z,t)-T_r\right]$, $\rho(z)$ is the density of materials of the heterostructure, δ_{ij} Is the Kronecker symbol. With an account, the relation for σ_{ij} last system of the equation could be written as,

$$\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_x(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}+$$

$$+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial z^2}\right]+\left[K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right]\frac{\partial^2 u_z(x,y,z,t)}{\partial x\partial z}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}$$

$$\rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial y}\right]-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}\times$$

$$\times\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_y(x,y,z,t)}{\partial z}+\frac{\partial u_z(x,y,z,t)}{\partial y}\right]\right\}+K(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}+ \quad (8)$$

$$+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}$$

$$\rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_z(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial z}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}\right]+$$

$$+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x}+\frac{\partial u_y(x,y,z,t)}{\partial y}+\frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}+$$

$$+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(x,y,z,t)}{\partial z}-\frac{\partial u_x(x,y,z,t)}{\partial x}-\frac{\partial u_y(x,y,z,t)}{\partial y}-\frac{\partial u_z(x,y,z,t)}{\partial z}\right]\right\}$$

Conditions for the system of (8) could be written in the form,

$$\frac{\partial \vec{u}(0,y,z,t)}{\partial x}=0;\quad \frac{\partial \vec{u}(L_x,y,z,t)}{\partial x}=0;\quad \frac{\partial \vec{u}(x,0,z,t)}{\partial y}=0;\quad \frac{\partial \vec{u}(x,L_y,z,t)}{\partial y}=0;$$

$$\frac{\partial \vec{u}(x,y,0,t)}{\partial z}=0;\quad \frac{\partial \vec{u}(x,y,L_z,t)}{\partial z}=0;\quad \vec{u}(x,y,z,0)=\vec{u}_0;\quad \vec{u}(x,y,z,\infty)=\vec{u}_0$$

We determine Spatio-temporal distributions of concentrations of dopant and radiation defects by solving the (1), (3), and (5) framework standard method of averaging function corrections [26]. Previously we transform the (1), (3), and (5) to the following form with an account of initial distributions of the considered concentrations,

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+f_C(x,y,z)\delta(t)+ \tag{1a}$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}C(x,y,W,t)dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}C(x,y,W,t)dW\right]$$

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]-$$

$$-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_I(x,y,z)\delta(t)+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right] \tag{3a}$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]-k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\right]-$$

$$-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_V(x,y,z)\delta(t)+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]$$

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+k_I(x,y,z,T)I(x,y,z,t)+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)dW\right]+k_I(x,y,z,T)I(x,y,z,t)+f_{\Phi_I}(x,y,z)\delta(t) \quad (5a)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+k_V(x,y,z,T)V(x,y,z,t)+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)dW\right]+k_V(x,y,z,T)V(x,y,z,t)+f_{\Phi_V}(x,y,z)\delta(t)$$

Farther we replace concentrations of dopant and radiation defects on the right sides of (1a), (3a), and (5a) on their not yet known average values $\alpha_{1\rho}$. In this situation, we obtain equations for the first-order approximations of the required concentrations in the following form,

$$\frac{\partial C_1(x,y,z,t)}{\partial t}=\alpha_{1C}\Omega\frac{\partial}{\partial x}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1C}\Omega\frac{\partial}{\partial y}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+f_C(x,y,z)\delta(t) \quad (1b)$$

$$\frac{\partial I_1(x,y,z,t)}{\partial t}=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+\alpha_{1I}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+f_I(x,y,z)\delta(t)-$$

$$-\alpha_{1I}^2k_{I,I}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T) \quad (3b)$$

$$\frac{\partial V_1(x,y,z,t)}{\partial t}=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1V}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+f_V(x,y,z)\delta(t)-$$

$$-\alpha_{1V}^2k_{V,V}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T)$$

$$\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+f_{\Phi_I}(x,y,z)\,\delta(t)+$$

$$+k_I(x,y,z,T)\,I(x,y,z,t)+k_{I,I}(x,y,z,T)\,I^2(x,y,z,t) \quad (5b)$$

$$\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+f_{\Phi_V}(x,y,z)\,\delta(t)+$$

$$+k_V(x,y,z,T)\,V(x,y,z,t)+k_{V,V}(x,y,z,T)\,V^2(x,y,z,t)$$

Integration of the left and right sides of the (1b), (3b), and (5b) on time gives us the possibility to obtain relations for approximation in the final form

$$C_1(x,y,z,t)=\alpha_{1C}\Omega\frac{\partial}{\partial x}\int\limits_0^t D_{SL}(x,y,z,T)\frac{z}{kT}\left[1+\frac{\xi_S\alpha_{1C}^\gamma}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\times\nabla_S\mu_1(x,y,z,\tau)\,d\,\tau+\alpha_{1C}\frac{\partial}{\partial y}\int_0^t D_{SL}(x,y,z,T)\left[1+\frac{\xi_S\alpha_{1C}^{\gamma}}{P^{\gamma}(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\times\,\Omega\nabla_S\mu_1(x,y,z,\tau)\frac{z}{kT}d\,\tau+f_C(x,y,z) \tag{1c}$$

$$I_1(x,y,z,t)=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\,\tau+\alpha_{1I}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\,\tau-\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)\,d\,\tau+$$

$$+f_I(x,y,z)-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\,\tau \tag{3c}$$

$$V_1(x,y,z,t)=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\,\tau+\alpha_{1V}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\,\tau-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)\,d\,\tau+$$

$$+f_V(x,y,z)-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\,\tau$$

$$\Phi_{1I}(x,y,z,t)=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\,\tau+\alpha_{1\Phi_I}z\,\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\,\tau+f_{\Phi_I}(x,y,z)+$$

$$+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)d\,\tau+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)d\,\tau \tag{5c}$$

$$\Phi_{1V}(x,y,z,t)=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\,\tau+\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\,\tau+f_{\Phi_V}(x,y,z)+$$

$$+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)d\,\tau+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)d\,\tau$$

We determine average values of the first-order approximations of concentrations of dopant and radiation defects by the following standard relation [26].

$$\alpha_{1\rho}=\frac{1}{\Theta L_xL_yL_z}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\rho_1(x,y,z,t)d\,z\,d\,y\,d\,x\,d\,t \tag{9}$$

Substitution of the relations (1*c*), (3*c*), and (5*c*) into relation (9) gives us the possibility to obtain the required average values in the following form

$$\alpha_{1C}=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_C(x,y,z)d\,z\,d\,y\,d\,x,$$

$$\alpha_{1I}=\sqrt{\frac{(a_3+A)^2}{4a_4^2}-4\left(B+\frac{\Theta a_3B+\Theta^2L_xL_yL_za_1}{a_4}\right)}-\frac{a_3+A}{4a_4},$$

$$\alpha_{1V}=\frac{1}{S_{IV00}}\left[\frac{\Theta}{\alpha_{1I}}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x-\alpha_{1I}S_{II00}-\Theta L_xL_yL_z\right],$$

Where,

$$S_{\rho\rho\, ij}=\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{\rho,\rho}(x,y,z,T)I_1^i(x,y,z,t)V_1^j(x,y,z,t)d\,z\,d\,y\,d\,x\,d\,t\,,\quad a_4=S_{II\,00}\left(S_{IV\,00}^2-S_{II\,00}S_{VV\,00}\right),$$

$$a_3=S_{IV\,00}S_{II\,00}+S_{IV\,00}^2-S_{II\,00}S_{VV\,00}\,,\quad a_2=S_{IV\,00}S_{IV\,00}^2\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_V(x,y,z)d\,z\,d\,y\,d\,x+2S_{VV\,00}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)d\,z\,d\,y\,d\,x\times$$

$$\times S_{II\,00}+S_{IV\,00}\Theta L_x^2L_y^2L_z^2-\Theta L_x^2L_y^2L_z^2S_{VV\,00}-S_{IV\,00}^2\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)d\,z\,d\,y\,d\,x\,,\quad a_1=S_{IV\,00}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)d\,z\,d\,y\,d\,x\,,$$

$$a_0=S_{VV\,00}\left[\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)d\,z\,d\,y\,d\,x\right]^2,\quad A=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}}\,,\quad B=\frac{\Theta a_2}{6a_4}+\sqrt[3]{\sqrt{q^2+p^3}-q}-\sqrt[3]{\sqrt{q^2+p^3}+q}\,,$$

$$q=\frac{\Theta^3a_2}{24a_4^2}\left(4a_0-\Theta L_xL_yL_z\frac{a_1a_3}{a_4}\right)-\Theta^2\frac{a_0}{8a_4^2}\left(4\Theta a_2-\Theta^2\frac{a_3^2}{a_4}\right)-\frac{\Theta^3a_2^3}{54a_4^3}-L_x^2L_y^2L_z^2\frac{\Theta^4a_1^2}{8a_4^2}\,,\quad p=\Theta^2\frac{4a_0}{a_4}-\frac{\Theta a_2}{18a_4}--\Theta^3L_xL_yL_za_1a_3/12a_4^2$$

$$\alpha_{1\Phi_I}=\frac{R_{I1}}{\Theta L_xL_yL_z}+\frac{S_{II\,20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_{\Phi_I}(x,y,z)\,d\,z\,d\,y\,d\,x$$

$$\alpha_{1\Phi_V}=\frac{R_{V1}}{\Theta L_xL_yL_z}+\frac{S_{VV\,20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_{\Phi_V}(x,y,z)\,d\,z\,d\,y\,d\,x\,,\quad R_{\rho\,i}=\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_I(x,y,z,T)I_1^i(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t$$

Where we determine approximations of the second and higher orders of concentrations of dopant and radiation defects framework standard iterative procedure of a method of averaging of function corrections [26]. Framework this procedure to determine approximations of the *n*-th order of concentrations of dopant and radiation defects we replace the required concentrations in the (1*c*), (3*c*), (5*c*) on the following sum $\alpha_{n\rho}+\rho_{n-1}(x,y,z,t)$. The replacement leads to the following transformation of the appropriate equations,

$$\frac{\partial C_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left(D_L(x,y,z,T)\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^{\gamma}}{P^{\gamma}(x,y,z,T)}\right\}\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\right.$$
$$\left.\times\frac{\partial C_1(x,y,z,t)}{\partial x}\right)+\frac{\partial}{\partial y}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\frac{\partial C_1(x,y,z,t)}{\partial y}\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^{\gamma}}{P^{\gamma}(x,y,z,T)}\right\}\times\right.$$
$$\left.\times D_L(x,y,z,T)\right)+\frac{\partial}{\partial z}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\frac{\partial C_1(x,y,z,t)}{\partial z}\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^{\gamma}}{P^{\gamma}(x,y,z,T)}\right\}\times\right.$$
$$\left.\times D_L(x,y,z,T)\right)+f_C(x,y,z)\delta(t)+\Omega\frac{\partial}{\partial x}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}[\alpha_{2C}+C(x,y,W,t)]\,d\,W\right\}+$$
$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}[\alpha_{2C}+C(x,y,W,t)]\,d\,W\right\} \tag{1d}$$

$$\frac{\partial I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\right]-k_{I,V}(x,y,z,T)\times$$
$$\times[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]-k_{I,I}(x,y,z,T)[\alpha_{1I}+I_1(x,y,z,t)]^2+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial z}\right]+$$
$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,d\,W\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,d\,W\right\} \tag{3d}$$

$$\frac{\partial V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\right]-k_{I,V}(x,y,z,T)\times$$

$$\times[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]-k_{I,I}(x,y,z,T)[\alpha_{1I}+I_1(x,y,z,t)]^2+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\right\}$$

$$\frac{\partial\Phi_{2I}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+k_I(x,y,z,T)I(x,y,z,t)+f_{\Phi_I}(x,y,z)\,\delta(t)+$$

$$+k_{I,I}(x,y,z,T)I^2(x,y,z,t) \tag{5d}$$

$$\frac{\partial\Phi_{2V}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+k_V(x,y,z,T)V(x,y,z,t)+f_{\Phi_V}(x,y,z)\,\delta(t)+$$

$$+k_{V,V}(x,y,z,T)V^2(x,y,z,t)$$

Integration of the left and the right sides of (1d), (3d), and (5d) gives us the possibility to obtain relations for the required concentrations in the final form

$$C_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\frac{\partial C_1(x,y,z,\tau)}{\partial x}\times$$

$$\times D_L(x,y,z,T)d\tau+\frac{\partial}{\partial y}\int_0^t D_L(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times$$

$$\times\frac{\partial C_1(x,y,z,\tau)}{\partial y}d\,\tau+\frac{\partial}{\partial z}\int_0^t\frac{\partial C_1(x,y,z,\tau)}{\partial y}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times$$

$$\times D_L(x,y,z,T)\,d\tau+f_C(x,y,z)+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_S}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}[\alpha_{2C}+C_1(x,y,W,\tau)]\,dW\,d\tau+$$

$$+\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_S}{kT}\nabla_S\mu\,(x,y,z,\tau)\int_0^{L_z}[\alpha_{2C}+C_1(x,y,W,\tau)]\,dW\,d\tau \tag{1e}$$

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial y}\int_0^t D_I(x,y,z,T)\times$$

$$\times\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau-\int_0^t\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]^2\times$$

$$\times k_{I,I}(x,y,z,T)\,d\tau+f_I(x,y,z)+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{IS}}{kT}\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\,d\tau+$$

$$+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{IS}}{kT}\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\,d\tau \tag{3e}$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial y}\int_0^t D_V(x,y,z,T)\times$$

$$\times\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau-\int_0^t\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]^2\times$$

$$\times k_{V,V}(x,y,z,T)\,d\tau+f_V(x,y,z)+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{VS}}{kT}\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau+$$

$$+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{VS}}{kT}\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau$$

$$\Phi_{2I}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial y}d\tau+$$

$$+\frac{\partial}{\partial z}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial z}d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\,d\tau+$$

$$+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\,d\tau+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)\,d\tau+$$

$$+\int_0^t k_{I,I}(x,y,z,T)\,I^2(x,y,z,\tau)\,d\tau+f_{\Phi_I}(x,y,z) \tag{5e}$$

$$\Phi_{2V}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial y}d\tau+$$

$$+\frac{\partial}{\partial z}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial z}d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\,d\tau+$$

$$+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\,d\tau+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\tau+$$

$$+\int_0^t k_{V,V}(x,y,z,T)\,V^2(x,y,z,\tau)\,d\tau+f_{\Phi_V}(x,y,z).$$

Average values of the second-order approximations of required approximations by using the following standard relation [26].

$$\alpha_{2\rho} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [\rho_2(x,y,z,t) - \rho_1(x,y,z,t)]\, d\,z\, d\,y\, d\,x\, d\,t \tag{10}$$

Substitution of the relations (1e), (3e), and (5e) into relation (10) gives us the possibility to obtain relations for required average values α 2ρ.

$\alpha 2C=0$, $\alpha 2\Phi I=0$, $\alpha 2\Phi V=0$,

$$\alpha_{2V} = \sqrt{\frac{(b_3+E)^2}{4b_4^2} - 4\left(F + \frac{\Theta a_3 F + \Theta^2 L_x L_y L_z b_1}{b_4}\right)} - \frac{b_3+E}{4b_4},$$

$$\alpha_{2I} = \frac{C_V - \alpha_{2V}^2 S_{VV00} - \alpha_{2V}\left(2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z\right) - S_{VV02} - S_{IV11}}{S_{IV01} + \alpha_{2V} S_{IV00}},$$

Where,

$$b_4 = \frac{S_{IV00}^2 S_{VV00} - S_{VV00}^2 S_{II00}}{\Theta L_x L_y L_z}, \quad b_3 = -\frac{S_{II00} S_{VV00}}{\Theta L_x L_y L_z}\left(2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z\right) + \frac{S_{IV00}^2}{\Theta L_x L_y L_z}\left(2S_{VV01} + S_{IV10} + \right.$$

$$\left. + \Theta L_x L_y L_z\right) - \frac{S_{IV00}^2 S_{IV10}}{\Theta^3 L_x^3 L_y^3 L_z^3} + \frac{S_{IV00} S_{VV00}}{\Theta L_x L_y L_z}\left(S_{IV01} + 2S_{II10} + S_{IV01} + \Theta L_x L_y L_z\right), \quad b_2 = \frac{S_{II00} S_{VV00}}{\Theta L_x L_y L_z}\left(S_{VV02} + S_{IV11} + C_V\right) + \left(2S_{VV01} - \right.$$

$$\left. - \Theta L_x L_y L_z - S_{IV10}\right)^2 + \frac{S_{IV01} S_{VV00}}{\Theta L_x L_y L_z}\left(\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}\right) + \frac{S_{IV00}}{\Theta L_x L_y L_z}\left(S_{IV01} + 2S_{II10} + 2S_{IV01} + \Theta L_x L_y L_z\right) \times$$

$$\times \left(2S_{VV01} + \Theta L_x L_y L_z + S_{IV10}\right) - \frac{S_{IV00}^2}{\Theta L_x L_y L_z}\left(C_V - S_{VV02} - S_{IV11}\right) + \frac{C_I S_{IV00}^2}{\Theta^2 L_x^2 L_y^2 L_z^2} - 2S_{IV10}\frac{S_{IV00} S_{IV01}}{\Theta L_x L_y L_z}, \quad b_1 = S_{II00}\left(2S_{VV01} + \right.$$

$$\left. + S_{IV10} + \Theta L_x L_y L_z\right)\frac{S_{IV11} + S_{VV02} + C_V}{\Theta L_x L_y L_z} + \frac{S_{IV01}}{\Theta L_x L_y L_z}\left(2S_{II10} + S_{IV01} + \Theta L_x L_y L_z\right)\left(2S_{VV01} + S_{IV01} + \Theta L_x L_y L_z\right) - \frac{S_{IV10}}{\Theta L_x} \times$$

$$\times \frac{S_{IV01}^2}{L_y L_z} - S_{IV00}\frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z}\left(3S_{IV01} + 2S_{II10} + \Theta L_x L_y L_z\right) + 2C_I S_{IV00} S_{IV01}, \quad b_0 = S_{II00}\frac{\left(S_{IV00} + S_{VV02}\right)^2}{\Theta L_x L_y L_z} - \frac{S_{IV01}}{L_x L_y L_z} \times$$

$$\times \frac{1}{\Theta}\left(\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}\right)\left(C_V - S_{VV02} - S_{IV11}\right) + 2C_I S_{IV01}^2 - S_{IV01}\frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z}\left(\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}\right)$$

$$C_I = \frac{\alpha_{1I}\alpha_{1V}}{\Theta L_x L_y L_z} S_{IV00} + \frac{\alpha_{1I}^2 S_{II00}}{\Theta L_x L_y L_z} - \frac{S_{II20} S_{II20}}{\Theta L_x L_y L_z} - \frac{S_{IV11}}{\Theta L_x L_y L_z}, \quad C_V = \alpha_{1I}\alpha_{1V} S_{IV00} + \alpha_{1V}^2 S_{VV00} - S_{VV02} - S_{IV11}, \quad F = \frac{\Theta a_2}{6a_4} +$$

$$+ \sqrt[3]{\sqrt{r^2+s^3} - r} - \sqrt[3]{\sqrt{r^2+s^3} + r}, \quad E = \sqrt{8y + \Theta^2\frac{a_3^2}{a_4^2} - 4\Theta\frac{a_2}{a_4}}, \quad r = \frac{\Theta^3 b_2}{24b_4^2}\left(4b_0 - \Theta L_x L_y L_z\frac{b_1 b_3}{b_4}\right) - \frac{\Theta^3 b_2^3}{54b_4^3} - b_0\frac{\Theta^2}{8b_4^2} \times$$

$$\times \left(4\Theta b_2 - \Theta^2\frac{b_3^2}{b_4}\right) - L_x^2 L_y^2 L_z^2\frac{\Theta^4 b_1^2}{8b_4^2}, \quad s = \Theta^2\frac{4b_0 b_4 - \Theta L_x L_y L_z b_1 b_3}{12b_4^2} - \frac{\Theta b_2}{18b_4}.$$

Farther we determine solutions of (8), i.e. components of the displacement vector. To determine the first-order approximations of the considered components framework method of averaging function corrections we replace the required functions on the right sides of the equations with their not yet known average values αi. The substitution leads to the following result.

$$\rho(z)\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x},\quad \rho(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y},$$
$$\rho(z)\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}$$

Integration of the left and the right sides of the relations on-time t leads to the following result,

$$u_{1x}(x,y,z,t)=u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta,$$
$$u_{1y}(x,y,z,t)=u_{0y}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^t\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta,$$
$$u_{1z}(x,y,z,t)=u_{0z}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^t\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta$$

Approximations of the second and higher orders of components of the displacement vector could be determined by using standard replacement of the required components on the following sums $\alpha_i+u_i(x,y,z,t)$ [26]. The replacement leads to the following result.

$$\rho(z)\frac{\partial^2 u_{2x}(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}+$$
$$+\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x\partial z}+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial z^2}\right]-\frac{\partial T(x,y,z,t)}{\partial x}\times$$
$$\times K(z)\beta(z)$$

$$\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\right]-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}+$$
$$+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_{1y}(x,y,z,t)}{\partial z}+\frac{\partial u_{1z}(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+$$
$$+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}$$

$$\rho(z)\frac{\partial^2 u_{2z}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2\,[1+\sigma(z)]}\left[\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial z}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}\right]+$$
$$+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}\Bigg]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_{1x}(x,y,z,t)}{\partial x}+\frac{\partial u_{1y}(x,y,z,t)}{\partial y}+\frac{\partial u_{1x}(x,y,z,t)}{\partial z}\right]\right\}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}+$$
$$+\frac{\partial}{\partial z}\left\{\frac{E(z)}{6[1+\sigma(z)]}\left[6\frac{\partial u_{1z}(x,y,z,t)}{\partial z}-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]\right\}.$$

Integration of the left and right sides of the relations on-time t leads to the following result,

$$u_{2x}(x,y,z,t)=\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times$$
$$\times\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial y^2}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\times$$
$$\times\frac{1}{1+\sigma(z)}+\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial z}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}-K(z)\frac{\partial}{\partial x}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$
$$\times\frac{\beta(z)}{\rho(z)}-\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$
$$\left.+\frac{\partial^2}{\partial z^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\right]\frac{E(z)}{2\rho(z)}-\frac{1}{\rho(z)}\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)d\tau\,d\vartheta+$$
$$+u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta$$

$$u_{2y}(x,y,z,t)=\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial x^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)d\tau\,d\vartheta+\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)d\tau\,d\vartheta\right]+$$
$$+\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)d\tau\,d\vartheta+\frac{1}{\rho(z)}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\frac{\partial^2}{\partial y^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{2\rho(z)}\times$$
$$\times\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\right]\right\}-K(z)\frac{\beta(z)}{\rho(z)}\int_0^t\int_0^\vartheta T(x,y,z,\tau)d\tau\,d\vartheta-$$
$$-\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{1}{\rho(z)}\frac{\partial^2}{\partial y\partial z}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)d\tau\,d\vartheta+\right.$$
$$-K(z)\frac{\beta(z)}{\rho(z)}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)d\tau\,d\vartheta-\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$
$$\times\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}-\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}-$$
$$-\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0y}$$

$$u_z(x,y,z,t)=\left[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial x\partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$
$$\left.+\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\frac{E(z)}{2\rho(z)[1+\sigma(z)]}-\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}+\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta-\right.\right.$$
$$\left.\left.-\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}\frac{1}{6\rho(z)}+u_{0z}-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta$$

In the framework of the paper, we determine the concentration of dopant, concentrations of radiation defects, and components of displacement vector by using the second-order approximation framework method of averaging function corrections. This approximation is usually enough good approximation to make a qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with the results of numerical simulations.

3. RESULTS AND DISCUSSION

In this section, we analyzed the dynamics of redistributions of dopant and radiation defects during annealing and under influence of mismatch-induced stress. Typical distributions of concentrations of dopant in heterostructures are presented in Figures 2 and 3 for diffusion and ion types of doping, respectively. These distributions have been calculated for the case when the value of the dopant diffusion coefficient in the epitaxial layer is larger than in the substrate. The figures show, that inhomogeneity of heterostructure gives us the possibility to increase the sharpness of *p-n-* junctions. At the same time, one can find increasing homogeneity of dopant distribution in the doped part of the epitaxial layer. Increasing of sharpness of the *p-n*-junction gives us the possibility to decrease switching time. The second effect leads to decreasing local heating of materials during the functioning of the *p-n*-junction or decreasing of dimensions of the *p-n*-junction for a fixed maximal value of local overheat. However, in the framework of this approach to the manufacturing of bipolar transistors, it is necessary to optimize the annealing of dopant and/or radiation defects. The reason for this optimization is the following. If the annealing time is small, the dopant did not achieve any interfaces between materials of the heterostructure. In this situation, one cannot find any modifications in the distribution of concentration of dopant. If the annealing time is large, the distribution of concentration of dopant is too homogenous. We optimize the annealing time framework recently introduces approach [15], [25]–[32]. To framework this criterion we approximate the real distribution of concentration of dopant by step-wise function as shown in Figure 4 and Figure 5. Farther we determine optimal values of annealing time by minimization of the following mean-squared error.

Where $\psi(x, y, z)$ is the approximation function. Dependences of optimal values of annealing time on parameters are presented in Figures 6 and 7 for diffusion and ion types of doping, respectively. It should be noted, that it is necessary to anneal radiation defects after ion implantation. One could find the spreading of the concentration of distribution of dopant during this annealing. In the ideal case distribution of dopant achieves appropriate interfaces between materials of heterostructure during the annealing of radiation defects. If the dopant did not achieve any interfaces during annealing of radiation defects, it is practicably to additionally anneal the dopant. In this situation, the optimal value of the additional annealing time of implanted dopant is smaller, than the annealing time of infused dopant.

Farther we analyzed the influence of relaxation of mechanical stress on the distribution of dopants in doped areas of the heterostructure. Under the following condition $\varepsilon_0 < 0$ one can find compression of distribution of concentration of dopant near the interface between materials of the heterostructure. Contrary (at $\varepsilon_0 > 0$) one can find a spreading of distribution of concentration of dopant in this area. This change of distribution of concentration of dopant could be at least partially compensated by using laser annealing [29]. This type of annealing gives us the possibility to accelerate the diffusion of dopants and other processes in the annealed area due to the inhomogenous distribution of temperature and Arrhenius law. Accounting relaxation of mismatch-induced stress in heterostructure could lead to changing of optimal values of annealing time. Mismatch-induced stress could be used to increase the density of elements of integrated circuits. On the other hand, could lead to generation dislocations of the discrepancy. Figure 8 shows distributions of components of the displacement vector, which is perpendicular to the interface between layers of the heterostructure.

$$U=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[C(x,y,z,\Theta)-\psi(x,y,z)\right]dz\,dy\,dx \tag{15}$$

Which is perpendicular to the interface between the epitaxial layer substrate. Increasing the number of curves corresponds to increasing of difference between values of the dopant diffusion coefficient in layers of heterostructure under conditions when the value of the dopant diffusion coefficient in the epitaxial layer is larger than the value of the dopant diffusion coefficient in the substrate.

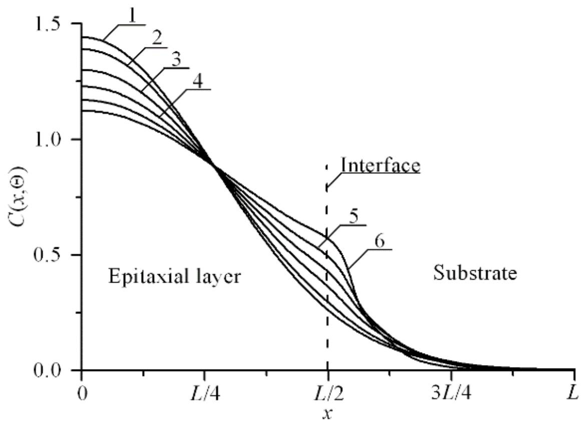

Figure 2. Distributions of concentration of infused dopant in heterostructure from Figure 1 in direction

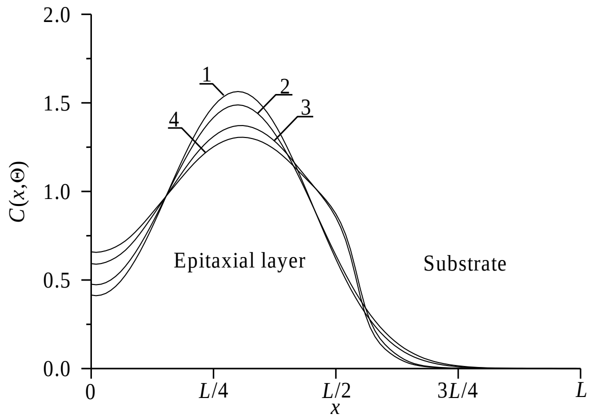

Figure 3. Distributions of concentration of implanted dopant in heterostructure from Figure 1 in direction

Which is perpendicular to the interface between the epitaxial layer substrate. Curves 1 and 3 corresponds to annealing time Θ = 0.0048(Lx2+Ly2+Lz2)/D0. Curves 2 and 4 corresponds to annealing time Θ = 0.0057(Lx2+Ly2+Lz2)/D0. Curves 1 and 2 correspond to a homogenous sample. Curves 3 and 4 correspond to heterostructure under the condition when the value of dopant diffusion coefficient in the epitaxial layer is larger than the value of dopant diffusion coefficient in the substrate.

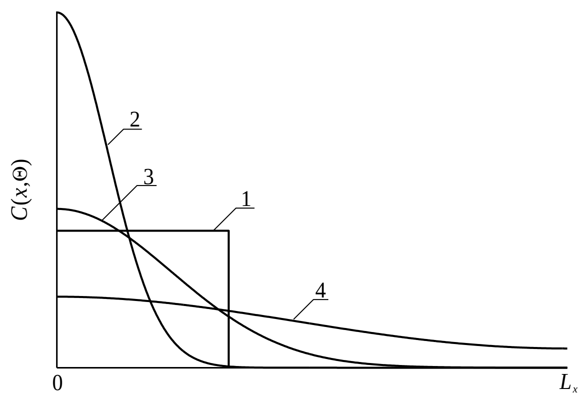

Figure 4. Spatial distributions of dopant in heterostructure after dopant infusion

Curve 1 is the idealized distribution of dopant. Curves 2-4 are real distributions of dopants for different values of annealing time. Increasing the number of curves corresponds to increasing of annealing time.

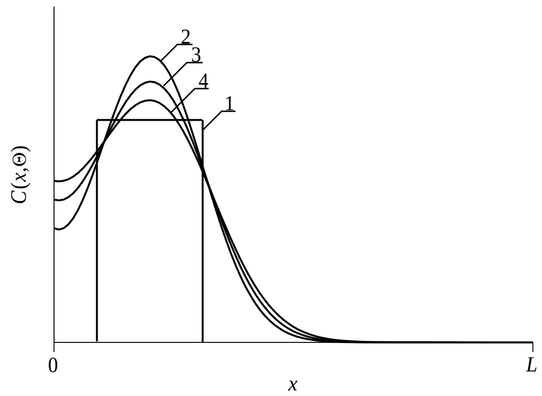

Figure 5. Spatial distributions of dopant in heterostructure after ion implantation

Curve 1 is the idealized distribution of dopant. Curves 2-4 are real distributions of dopants for different values of annealing time. Increasing the number of curves corresponds to an increase in annealing time.

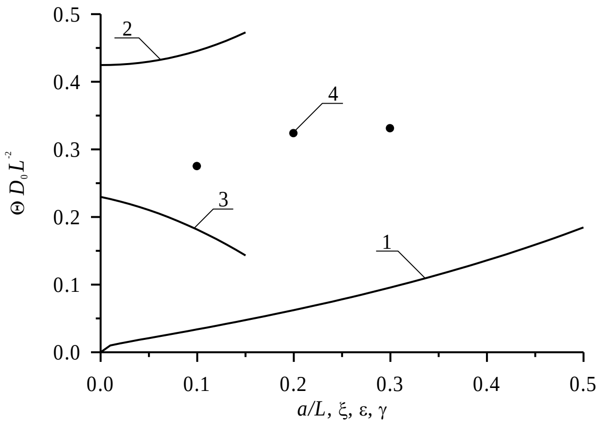

Figure 6. Dependences of dimensionless optimal annealing time for doping by diffusion

Which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for a/L=1/2 and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for a/L=1/2 and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for a/L=1/2 and $\varepsilon = \xi = 0$.

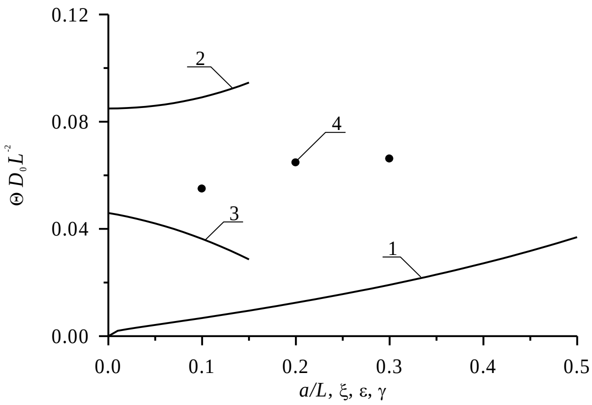

Figure 7. Dependences of dimensionless optimal annealing time for doping by ion implantation

Which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for a/L=1/2 and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for a/L=1/2 and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for a/L=1/2 and $\varepsilon = \xi = 0$.

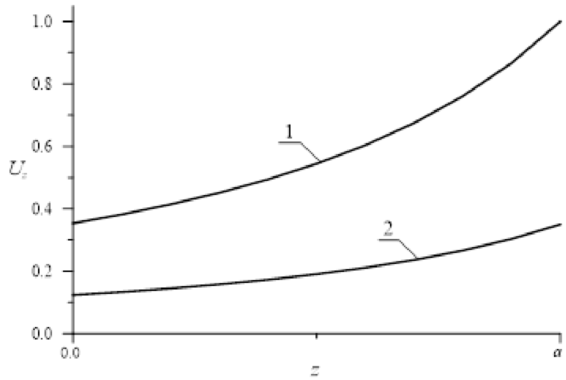

Figure 8. Normalized dependences of component uz of displacement vector on coordinate z for nonporous (curve 1) and porous (curve 2) epitaxial layers

4. CONCLUSION

In this paper, we model redistribution of infused and implanted dopants with account relaxation mismatch-induced stress during manufacturing field-effect heterotransistors in the framework of a nulling-resistor output amplifier. We formulate recommendations for optimization of annealing to decrease dimensions of the considered transistors and to increasing of their density. In this paper, we formulate a recommendation to decrease mismatch-induced stress by using radiation processing of stressed areas of the heterostructure. This effect was recently found experimentally, but in this paper, we introduce a model of this effect and an analytical approach to the prognosis of this effect. We also introduce an analytical approach for prognosis of diffusion and ion types of doping with account concurrent changing of parameters in space and time has. At the same time, the approach gives us the possibility to take into account the nonlinearity of considered processes. The considered approach is the significantly more common approach in comparison with other ones for related applications. At the same time, analytical approaches to the solution of boundary problems give explicit functional dependences of obtained solutions on all parameters in comparison with numerical ones.

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