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Adaptive fuzzy sliding mode controller for a single-stage inverted pendulum

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ABSTRACT

Sliding mode controller (SMC) has become a popular traditional control method in industries due to the most effective control strategies employing nonlinear control and the ability to reject disturbances, particularly for line trajectory control. However, this control method has chattering problems due to high-frequency switching. To cope with the shortcoming, an artificial intelligence (AI) method is utilized in the traditional SMC to eliminate or reduce this chattering problem. This paper investigates an adaptive fuzzy logic system combined with SMC algorithm to alleviate the problem. Fuzzy logic is chosen due to its advantages in tackling nonlinear properties using if-then thinking, whereas SMC method can be applied due to its ability to reject disturbance control. The inverted pendulum is selected as a controlled object and simulated using MATLAB/Simulink to investigate this control method. By combining the fuzzy logic system and the SMC approach, the chattering problems can be adaptively reduced on the line trajectory tracking signal. The adaptive fuzzy SMC achieved better performance with fast response compared with previous literature algorithms for similar plants.

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1. INTRODUCTION

In the last decade, many scholars have investigated several control methods to achieve control optimization [1], [2]. This strategy is carried out to find new control ideas or modify them by combining two or more control algorithms to reduce or eliminate shortcomings. The combination of these algorithms can be conducted between conventional control and modern control strategies. By this combination, an adaptive control strategy can be obtained by online estimation to approximate the parameter varies corresponding to the unknown model [3]. For example, the proportional-integral-derivative (PID) control approach is combined with the artificial intelligence (AI) method to achieve adaptive intelligent control [4]. Similar kind to PID controller, the sliding mode controller (SMC) is also possibly be combined with AI approach to improving the control performance [5]. Moreover, traditional control systems usually employ AI algorithms to obtain a more optimal and robust control system [6].

The control system algorithm is generally divided into three functions: line tracking control, balance and velocity control, and steering control [7]. The traditional SMC control algorithm is very popular to be applied to line tracking control because of its ability to reject disturbance control [8]. According to the researches on [9]–[11], this control algorithm is one of the most effective control strategies to tackle the system with nonlinear properties due to its robustness in parameter variations and external disturbances. However, due to high-frequency switching, the traditional SMC control algorithm has a shortcoming, namely chattering phenomena on the sliding surface [12]. The chattering problem is an undesired phenomenon of oscillations and becomes a problem in obtaining better control accuracy [13]. To eliminate or reduce this chattering problem, combining different characteristics of control methods are become a solution to achieve the tracking control problem effectively and accurately.

Various AI methods have been studied to overcome the chattering problems of the SMC algorithm, including the fuzzy method, and combining it with other algorithms, such as heuristic or neural network strategy. One of these methods employed a neural network to tackle the problem. Cibiraj and Varatharajan [14], the chattering phenomenon can be efficiently reduced using an adaptive neural gain scheduling SMC approach and eliminated using adaptive fuzzy gain scheduling SMC. Li *et al.* [15], the adaptive neural network method is employed to effectively alleviate chattering for a class of nonlinear single-input–single-output (SISO) systems. Yogi *et al.* [16] employed a recurrent neural network algorithm to attenuate the chattering effect on SMC for controlling the position of the quadrotor. This study in [16] could effectively attenuate the chattering effect on SMC and provide faster and finite-time convergence. Moreover, Cheng *et al.* [17] suppressed the chattering problem on SMC algorithm for flexible-joint robot manipulator using radial basis function neural network (RBFNN). This study obtained effectively eliminate and suppressed the chattering problem with smaller switching compared with other algorithms without RBFNN.

Another way to reduce or eliminate the chattering on the traditional SMC is combined with fuzzy logic control [18]. Baghaei *et al.* [19] proposed a chattering-free (CF) SMC, namely the CFSMC approach, using a fuzzy logic model to eliminate the chattering problem. This study achieved less computational cost, better stability, and high accuracy control system. Ahmed *et al.* [20] studied modeling and chattering free for SMC using an adaptive fuzzy algorithm employed on the robotic manipulator systems. The proposed method achieved a chattering-free fuzzy SMC for uncertain chaotic systems. An adaptive interval type-2 fuzzy SMC has also been presented to avoid the chattering problem for a robot arm to validate the practical control approach [21]. As reported in [22], the fuzzy logic system is also adapted to offset the errors on uncertain nonlinear systems. Most studies above proved that a fuzzy logic system could better reduce and alleviate the SMC method's chattering problems.

The contribution of this study is to design a control strategy using a combination of two algorithms, such as fuzzy logic and SMC method, as an alternative for a single-stage inverted pendulum. By combining an adaptive fuzzy control and the SMC method, a new control strategy can enhance control accuracy and robustness by reducing the chattering phenomenon. This proposed algorithm is then called adaptive fuzzy SMC (AFSMC). This proposed control study employs and simulates the controlled object using a single-stage inverted pendulum. The control performance of adaptive fuzzy combining with traditional SMC can be simulated using MATLAB/Simulink.

2. RESEARCH METHOD

2.1. System description

This paper considers the following n-order nonlinear SISO systems as the controlled object [23], [24]. This controlled object employed an inverted pendulum as representative of a nonlinear SISO system. This inverted pendulum is employed in the system due to the most important problems in the control laboratories to verify a modern control theory [25]. There is the most familiar inverted pendulum in literature. In this study, we employed a simulated cart inverted pendulum using MATLAB/Simulink. This dynamic equation can be modeled in the canonical form as (1).

$$\begin{aligned} x^{(n)} &= f(\mathbf{x},t) + g(\mathbf{x},t)\mathrm{u}(t) + \mathrm{d}(t) \\ y &= x \end{aligned} \tag{1}$$

Where f and g are unknown functions, respectively. $\mathrm{x} \in \mathbf{R}^{\mathrm{n}}, \mathrm{u} \in \mathbf{R}^{\mathrm{n}}, \mathrm{y} \in \mathbf{R}^{\mathrm{n}}$, and $\mathrm{d}(t)$ are unknown disturbances, $|d(t)| \le D, g(\mathbf{x},t) \ne 0, g(\mathbf{x},t) > 0$. If $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ are unknown values, the fuzzy systems $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ can be used instead of $f(\mathbf{x},t)$ and $g(\mathbf{x},t)$ to achieve the adaptive fuzzy SMC method.

2.2. Basic fuzzy system

Fuzzy control is a kind of control strategy that does not trust the controlled model but relies heavily on an expert or a knowledgeable control operator [26]. The advantage of a fuzzy control system is that it is easy to incorporate the experience or expertise of a human into the controller. Without this control experience, designing a high-level fuzzy control system becomes a challenging issue. In addition, because the fuzzy controller adopts if-then control rules, it is challenging issue to alter the control parameters, so control stability is also difficult to be achieved. In some complex systems, especially where the system has inaccurate and uncertain information, the effect of fuzzy control is often better than conventional control. The fuzzy controller parameters must be determined after repeated trial and error, and there is a lack of systematic analysis and comprehensive methods such as stability analysis. Let the system consist of fuzzy rules in the form of if-then and it can be written as (2).

$$\mathrm{R}^{(j)}: \text{ if } x_1 \text{ is } \mathrm{A}_1^j \text{ and ..... and } x_n \text{ is } \mathrm{A}_n^j \text{ then } y \textit{ is } \mathrm{B}^j \tag{2}$$

Using the inference engine, the output of the fuzzy system can be calculated using (3).

$$y(x) = \frac{\sum_{j=1}^{m} y^j \left(\prod_{i=1}^{n} \mu_{A_i^j}(x_i) \right)}{\sum_{j=1}^{m} \left(\prod_{i=1}^{n} \mu_{A_i^j}(x_i) \right)} \tag{3}$$

Where $\mu_{A_i^j}(x_i)$ is the fuzzy membership function, the (3) can be written as (4) and (5).

$$y(x) = \boldsymbol{\theta}^T \xi(x) \tag{4}$$

$$\boldsymbol{\theta} = [y^1 \quad \mathrm{L} \quad y^m]^T, \; \xi(x) = [\xi^1(x) \quad \mathrm{L} \quad \xi^m(x)]^T \tag{5}$$

Where $\xi(x)$ can be written as (6).

$$\xi(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^j}(x_i)}{\sum_{j=1}^{m} \left(\prod_{i=1}^{n} \mu_{A_i^j}(x_i) \right)} \tag{6}$$

2.3. Design of AFSMC algorithm

Designing an adaptive control system is an essential way to obtain good control performance. It is intended that the appropriate control method is also regarded by using appropriate methods or theories. The trial-and-error method is one of the strategies for designing a control system, yet this method requires a lot of time and energy to obtain a good control system. Therefore, if adaptive control law theory is available, why not use it to obtain reliable and reasonable control results. Considering the advantages and disadvantages of the algorithm, we choose an appropriate control algorithm or combine it with another one. The main goal of this strategy is to obtain better control performance by eliminating the drawback of the algorithm.

Here is one of the control strategies to reduce the chattering problem using a fuzzy logic system on the SMC system. The following is a control system design using an algorithm with good stability analysis. Designing an adaptive fuzzy SMC is firstly written the switching function of the SMC as (7).

$$s(x,t) = -\mathbf{Ke} = -(k_1 e + k_2 \dot{e} + \cdots + k_{n-1} e^{(n-2)} + e^{(n-1)}) \tag{7}$$

Where $\mathbf{e} = \mathbf{x}_d - \mathbf{x} = [e, \dot{e}, \ldots, e^{n-1}], k_1, \ldots, k_{n-1}$. The control law can be generated as (8).

$$u(t) = \frac{1}{g(\mathbf{x},t)} \left(\sum_{i=1}^{n-1} k_i e^{(i)} + x_d^{(n)} - d(t) - u_{sw} - f(\mathbf{x},t) \right) \tag{8}$$

Where $u_{SW} = \eta \operatorname{sgn}(s)$ and $\eta > 0$. By inserting (8) into (1), $\dot{s}(\mathbf{x},t)$ can be formulated as (9)

$$\dot{s}(\mathbf{x},t) = -u_{SW} = -\eta \operatorname{sgn}(s) \tag{9}$$

Then $s(\mathbf{x},t) \cdot \dot{s}(\mathbf{x},t) = -\eta |s| \le 0$

Where *f*, *g*, and *d* are unknown values, the control law in (8) is not possible to be applied on the system. These values could be approximated using fuzzy logic systems notated by $\hat{f}$, $\hat{g}$, and $\hat{h}$. The product inference engine uses the single-value-fuzzifier and the central-average-defuzzifier. The outputs of the fuzzy are $\hat{f}$, $\hat{g}$, and $\hat{h}$, respectively. Hence, the control law of the system can be rewritten as (10).

$$u(t)=\frac{1}{\hat{g}(\mathbf{x},t)}\left[\sum_{i=1}^{n-1}\mathbf{k}_i\mathbf{e}^{(i)}+x_d^{(n)}-\hat{h}(s)-\hat{f}(\mathbf{x},t)\right] \quad (10)$$

The approximation of $\hat{f}$, $\hat{g}$, and $\hat{h}$ can be generated. Those approximation values can be calculated using (11).

$$\begin{aligned}\hat{f}(x\,|\,\boldsymbol{\theta}_f)&=\boldsymbol{\theta}_f^{\mathrm{T}}\boldsymbol{\xi}(x)\\ \hat{g}(x\,|\,\boldsymbol{\theta}_g)&=\boldsymbol{\theta}_g^{\mathrm{T}}\boldsymbol{\xi}(x)\\ \hat{h}(s\,|\,\boldsymbol{\theta}_h)&=\boldsymbol{\theta}_h^{\mathrm{T}}\phi(s)\end{aligned} \quad (11)$$

Where $\hat{f}(x\,|\,\boldsymbol{\theta}_f)$, $\hat{g}(x\,|\,\boldsymbol{\theta}_g)$, and $\hat{h}(x\,|\,\boldsymbol{\theta}_h)$ are the fuzzy approximation of the (3), $\phi(s)$ and $\boldsymbol{\xi}(x)$ are the fuzzy vectors of the (5), vector $\boldsymbol{\theta}_f^{\mathrm{T}},\boldsymbol{\theta}_g^{\mathrm{T}}$ and $\boldsymbol{\theta}_h^{\mathrm{T}}$ varies according to the adaptive law, and $\hat{h}(x\,|\,\boldsymbol{\theta}_h)$ can be expressed using equations as (12), (13), and (14).

$$\hat{h}(s\,|\,\theta_h^*)=\eta_\Delta\,\mathrm{sgn}(s) \quad (12)$$

$$\eta_\Delta=D+\eta\ ,\ \eta\geq 0 \quad (13)$$

$$|\,d(t)\,|\leq 0 \quad (14)$$

Design adaptive law can be obtained by finding the solution of derivative vector $\boldsymbol{\theta}_f^{\mathrm{T}}$, $\boldsymbol{\theta}_g^{\mathrm{T}}$, and $\boldsymbol{\theta}_h^{\mathrm{T}}$. They can be written as (15), (16), and (17).

$$\dot{\boldsymbol{\theta}}_f=r_1 s\boldsymbol{\xi}(\mathbf{x}) \quad (15)$$

$$\dot{\boldsymbol{\theta}}_g=r_2 s\boldsymbol{\xi}(\mathbf{x})u \quad (16)$$

$$\dot{\boldsymbol{\theta}}_h=r_3 s\phi(s) \quad (17)$$

The optimal parameters of the vector $\boldsymbol{\theta}_f^T$, $\boldsymbol{\theta}_g^T$, and $\boldsymbol{\theta}_h^T$ can be defined using equations as (18), (19), and (20).

$$\boldsymbol{\theta}_f^*=\arg\min_{\boldsymbol{\theta}_f\in\Omega_f}\left(sub\,|\sup_{x\in\mathbf{R}^n}|\,\hat{f}(\mathbf{x}\,|\,\boldsymbol{\theta}_f)-f(\mathbf{x},t)\,|\right) \quad (18)$$

$$\boldsymbol{\theta}_g^*=\arg\min_{\boldsymbol{\theta}_g\in\Omega_g}\left(sub\,|\sup_{x\in\mathbf{R}^n}|\,\hat{g}(\mathbf{x}\,|\,\boldsymbol{\theta}_g)-g(\mathbf{x},t)\,|\right) \quad (19)$$

$$\boldsymbol{\theta}_h^*=\arg\min_{\boldsymbol{\theta}_h\in\Omega_h}\left(sub\,|\sup_{x\in\mathbf{R}^n}|\,\hat{h}(\mathbf{x}\,|\,\boldsymbol{\theta}_h)-u_{sw}\,|\right) \quad (20)$$

Where the symbols Ω_f,Ω_g and Ω_h can be set of the vectors $\boldsymbol{\theta}_f,\boldsymbol{\theta}_g$ and $\boldsymbol{\theta}_h$. The approximation error values of the control system can be calculated as (21) and (22).

$$\omega = f(\mathbf{x},t) + [g(\mathbf{x},t) - \hat{g}(x \mid \boldsymbol{\theta}_g^*)]\mathrm{u} - \hat{f}(\mathrm{x} \mid \boldsymbol{\theta}_f^*) \tag{21}$$

$$|\,\omega\,| \le \omega_{\max} \tag{22}$$

Then, the derivative $\dot{\mathrm{s}}$ can be derived as (23).

$$\begin{aligned}
\dot{\mathrm{s}} &= -\sum_{i=1}^{n} k_i e^{(\mathrm{i})} = -\sum_{i=1}^{n-1} k_i e^{(\mathrm{i})} - x_{\mathrm{d}}^{(\mathrm{n})} + x^{(\mathrm{n})} = -\sum_{i=1}^{n-1} k_i e^{(\mathrm{i})} + g(\mathbf{x},t)u(t) + f(\mathbf{x},t) + d(t) - x_{\mathrm{d}}^{(\mathrm{n})} \\
&= -\sum_{i=1}^{n-1} k_i e^{(\mathrm{i})} + [g(\mathbf{x},t) - \hat{g}(\mathbf{x},t)]u(t) + \hat{g}(\mathbf{x},t)u(t) + f(\mathbf{x},t) + d(t) - x_{\mathrm{d}}^{(\mathrm{n})} \\
&= f(\mathbf{x},t) + (g(\mathbf{x},t) - \hat{h}(s \mid \boldsymbol{\theta}_h) - \hat{f}(\mathbf{x},t) - \hat{g}(\mathbf{x},t))u(t) + d(t) \\
&= \hat{f}(\mathbf{x} \mid \boldsymbol{\theta}_f^*) + [g(\mathbf{x} \mid \boldsymbol{\theta}_g^*) - \hat{g}(\mathbf{x},t)]u(t) + d(t) + \omega + h(s \mid \boldsymbol{\theta}_h^*) - h(s \mid \boldsymbol{\theta}_g^*) - \hat{f}(\mathbf{x},t) - \hat{h}(s \mid \boldsymbol{\theta}_h) \\
&= \boldsymbol{\varphi}_f^{\mathrm{T}} \boldsymbol{\xi}(x) + \boldsymbol{\varphi}_g^{\mathrm{T}} \boldsymbol{\xi}(x)u(t) + \boldsymbol{\varphi}_h^{\mathrm{T}} \phi(s) + \mathrm{d}(t) + w - h(s \mid \boldsymbol{\theta}_h^*)
\end{aligned} \tag{23}$$

Where $\mathbf{k}_n = 1$, $\boldsymbol{\varphi}_f = \boldsymbol{\theta}_f^* - \boldsymbol{\theta}_f$, $\boldsymbol{\varphi}_g = \boldsymbol{\theta}_g^* - \boldsymbol{\theta}_g$, $\boldsymbol{\varphi}_h = \boldsymbol{\theta}_h^* - \boldsymbol{\theta}_h$. Defining the Lyapunov function as (24).

$$\mathrm{V} = 0.5\left(\mathrm{s}^2 + \mathrm{r}_1^{-1}\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{\varphi}_{\mathrm{f}} + \mathrm{r}_2^{-1}\boldsymbol{\varphi}_{\mathrm{g}}^{\mathrm{T}}\boldsymbol{\varphi}_{\mathrm{g}} + \mathrm{r}_3^{-1}\boldsymbol{\varphi}_{\mathrm{h}}^{\mathrm{T}}\boldsymbol{\varphi}_{\mathrm{h}}\right) \tag{24}$$

Where r_1, r_2 and r_3 are normal numbers. Then (23) can be derived as (25).

$$\begin{aligned}
\dot{\mathrm{V}} &= \mathrm{s}\dot{\mathrm{s}} + \frac{1}{\mathrm{r}_1}\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}\dot{\boldsymbol{\varphi}}_{\mathrm{f}} + \frac{1}{\mathrm{r}_2}\boldsymbol{\varphi}_{\mathrm{g}}^{\mathrm{T}}\dot{\boldsymbol{\varphi}}_{\mathrm{g}} + \frac{1}{\mathrm{r}_3}\boldsymbol{\varphi}_{\mathrm{h}}^{\mathrm{T}}\dot{\boldsymbol{\varphi}}_{\mathrm{h}} \\
&= \mathrm{s}\left[\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{\xi}(\mathbf{x}) + \boldsymbol{\varphi}_{\mathrm{g}}^{\mathrm{T}}\boldsymbol{\xi}(\mathbf{x})\mathrm{u}(\mathrm{t}) + \boldsymbol{\varphi}_{\mathrm{h}}^{\mathrm{T}}\phi(\mathrm{s}) + \omega + \mathrm{d}(\mathrm{t}) - \hat{\mathrm{h}}(\mathrm{s} \mid \theta_{\mathrm{h}}^*)\right] + \frac{1}{\mathrm{r}_1}\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}\dot{\boldsymbol{\varphi}}_{\mathrm{f}} + \frac{1}{\mathrm{r}_2}\boldsymbol{\varphi}_{\mathrm{g}}^{\mathrm{T}}\dot{\boldsymbol{\varphi}}_{\mathrm{g}} + \frac{1}{\mathrm{r}_3}\boldsymbol{\varphi}_{\mathrm{h}}^{\mathrm{T}}\dot{\boldsymbol{\varphi}}_{\mathrm{h}} \\
&= \mathrm{s}\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}\boldsymbol{\xi}(\mathbf{x}) + \frac{1}{\mathrm{r}_1}\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}\dot{\boldsymbol{\varphi}}_{\mathrm{f}} + \mathrm{s}\boldsymbol{\varphi}_{\mathrm{g}}^{\mathrm{T}}\boldsymbol{\xi}(\mathbf{x})\mathrm{u}(\mathrm{t}) + \frac{1}{\mathrm{r}_2}\boldsymbol{\varphi}_{\mathrm{g}}^{\mathrm{T}}\dot{\boldsymbol{\varphi}}_{\mathrm{g}} + \mathrm{s}\boldsymbol{\varphi}_{\mathrm{h}}^{\mathrm{T}}\phi(\mathrm{s}) + \frac{1}{\mathrm{r}_3}\boldsymbol{\varphi}_{\mathrm{h}}^{\mathrm{T}}\dot{\boldsymbol{\varphi}}_{\mathrm{h}} + \mathrm{s}[\mathrm{d}(\mathrm{t}) - \hat{\mathrm{h}}(\mathrm{s} \mid \boldsymbol{\theta}_{\mathrm{h}}^*)] + \mathrm{s}\omega
\end{aligned} \tag{25}$$

Because $\hat{\mathrm{h}}(\mathrm{s} \mid \boldsymbol{\theta}_{\mathrm{h}}^*)] = \eta_\Delta \,\mathrm{sgn}(\mathrm{s})$, then, (24) can be rewritten as (26).

$$\begin{aligned}
\dot{\mathrm{V}} &= \frac{1}{\mathrm{r}_1}\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}[\mathrm{r}_1\mathrm{s}\boldsymbol{\xi}(\mathbf{x}) + \dot{\boldsymbol{\varphi}}_{\mathrm{f}}] + \frac{1}{\mathrm{r}_2}\boldsymbol{\varphi}_{\mathrm{g}}^{\mathrm{T}}[\mathrm{r}_2\mathrm{s}\boldsymbol{\xi}(\mathbf{x})\mathrm{u}(\mathrm{t}) + \dot{\boldsymbol{\varphi}}_{\mathrm{g}}] + \frac{1}{\mathrm{r}_3}\boldsymbol{\varphi}_{\mathrm{h}}^{\mathrm{T}}[\mathrm{r}_3\mathrm{s}\phi(\mathrm{s}) + \dot{\boldsymbol{\varphi}}_{\mathrm{h}}] + \mathrm{sd}(\mathrm{t}) + \mathrm{s}\omega - (\mathrm{D} + \mu)\,|\,\mathrm{s}\,| \\
&\le \frac{1}{\mathrm{r}_1}\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}[\mathrm{r}_1\mathrm{s}\boldsymbol{\xi}(\mathbf{x}) + \dot{\boldsymbol{\varphi}}_{\mathrm{f}}] + \frac{1}{\mathrm{r}_2}\boldsymbol{\varphi}_{\mathrm{g}}^{\mathrm{T}}[\mathrm{r}_2\mathrm{s}\boldsymbol{\xi}(\mathbf{x})\mathrm{u}(\mathrm{t}) + \dot{\boldsymbol{\varphi}}_{\mathrm{g}}] + \frac{1}{\mathrm{r}_3}\boldsymbol{\varphi}_{\mathrm{h}}^{\mathrm{T}}[\mathrm{r}_3\mathrm{s}\phi(\mathrm{s}) + \dot{\boldsymbol{\varphi}}_{\mathrm{h}}] + \mathrm{s}\omega - \eta\,|\,\mathrm{s}\,|
\end{aligned} \tag{26}$$

Where $\dot{\phi}_{\mathrm{f}} = -\dot{\boldsymbol{\theta}}_{\mathrm{f}}$, $\dot{\phi}_{\mathrm{g}} = -\dot{\boldsymbol{\theta}}_{\mathrm{g}}$, $\dot{\phi}_{\mathrm{h}} = -\dot{\boldsymbol{\theta}}_{\mathrm{h}}$. Substituting (15)-(17) form into (26), then

$$\dot{\mathrm{V}} \le \mathrm{s}\omega - \eta\,|\,\mathrm{s}\,| \tag{27}$$

By fuzzy approach, the adaptive control can be realized by making the error ω approaching zero value. Therefore, it can be calculated using equation as (28).

$$\dot{\mathrm{V}} \le 0 \tag{28}$$

The proposed method using the adaptive fuzzy logic system with the SMC approach achieves a stable control system. The diagram block of adaptive fuzzy SMC can be described in Figure 1.

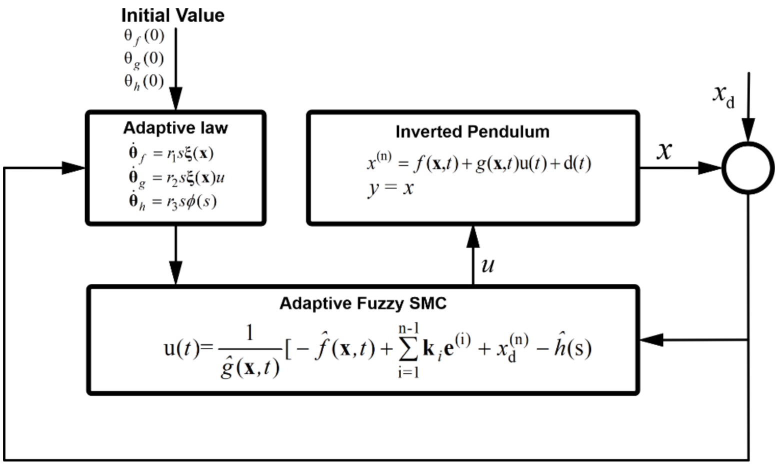


Figure 1. The adaptive fuzzy SMC for inverted pendulum

3. RESULTS AND DISCUSSION

In this part, we investigate two examples to verify the performance of the proposed fuzzy SMC algorithm in reducing chattering problems. We employed a single-stage inverted pendulum to verify the control tracking performance as the controlled object. In this paper, we employed the inverted pendulum as the controlled object written in (29). This simulation is conducted using MATLAB/Simulink as the control law (10). The controlled object takes a single-stage inverted pendulum and it can be described in Figure 2. The equation of this plant can be written as (29).

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + g(x) \cdot u \end{aligned} \tag{29}$$

Where

$$f(\mathbf{x}) = \frac{g \sin x_1 - mlx_2^2 \sin x_1 \cos x_1 / (m + m_c)}{l[4/3 - m\cos^2 x_1 / (m + m_c)]} \tag{30}$$

$$g(\mathbf{x}) = \frac{\cos x_1 / (m + m_c)}{l[4/3 - m\cos^2 x_1 / (m + m_c)]^u} \tag{31}$$

where $\mathbf{x} = [x_1, x_2]$ x_1 is the swing angle, x_2 is the swing speed, m is the pendulum mass, $\mathrm{m_c}$ is the trolley mass, g is the gravity velocity, l is the length, $g = 9.8\,\mathrm{m/s^2}$, $\mathrm{m} = 0.1\ \mathrm{kg}$, $\mathrm{m_c} = 1\ \mathrm{kg}$, l = 0. 5 m, and u is the input of the control.

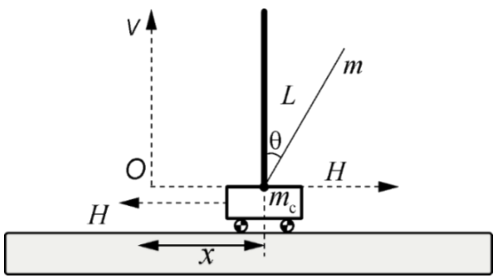


Figure 2. A single-stage inverted pendulum as the controlled object

The m/s position command as the reference signal is $\mathrm{x_d}(t) = 0.1\ \sin\ (\pi t)$, the function can be calculated using switching strategy as $s = -k_1 e - \dot{e}$, $k_1 = 5$. In this paper, we use five rules to approximate u(t). The membership functions, both s(t) and u(t), use "negative big" (NB), "negative small" (NS), "zero" (ZO), "positive small" (PS), and "positive big" (PB). The following five memberships u(t) are used to the fuzzy sliding surface and can be written as (32)-(36).

$$\mu_{NB}(x_i) = \exp\left[-[(\tfrac{x_i+\pi}{6})/(\tfrac{\pi}{24})]^2\right] \tag{32}$$

$$\mu_{NS}(x_i) = \exp\left[-[(\tfrac{x_i+\pi}{12})/(\tfrac{\pi}{24})]^2\right] \tag{33}$$

$$\mu_{ZO}(x_i) = \exp\left[-[(x_i)/(\tfrac{\pi}{24})]^2\right] \tag{34}$$

$$\mu_{PS}(x_i) = \exp\left[-[(\tfrac{x_i-\pi}{12})/(\tfrac{\pi}{24})]^2\right] \tag{35}$$

$$\mu_{PB}(x_i) = \exp\left[-[(\tfrac{x_i-\pi}{6})/(\tfrac{\pi}{24})]^2\right] \tag{36}$$

The membership u(t) of the adaptive SMC with the fuzzy method can be expressed in Figure 3. The switching function $s(t)$ can be converted using the fuzzy membership function as (37)-(39).

$$\mu_{NB}(s) = \exp(5(s+3))^{-1} + 1 \tag{37}$$

$$\mu_{ZO}(s) = \exp(-s^2) \tag{38}$$

$$\mu_{PB}(s) = \exp(5(s-3))^{-1} + 1 \tag{39}$$

We use 25 fuzzy rules to approximate both variable f and g values, and three fuzzy rules to approximate $s(t)$. The parameters setting of the simulation can be adjusted as follows. Let $\boldsymbol{\theta}_f^T$ and $\boldsymbol{\theta}_g^T$ be 25×1 vector, and $\boldsymbol{\theta}_h^T$ be 3×1 vector. Vector $\boldsymbol{\theta}_f^T$, $\boldsymbol{\theta}_g^T$ and $\boldsymbol{\theta}_h^T$ have an initial value 0.10. With the control law (9), the initial state is $[\pi/60, 0]$. The adaptive fuzzy parameters are defined as: $r_1 = 5$, $r_2 = 1$, and $r_3 = 10$. fsd_1, fsu_1, and fs_1 are used to represent the numerator $\xi_1(x)$, and denominator $\xi_1(x)$, whereas fsd_2, fsu_2, and fs_1 are denoted the numerator $\xi_2(x)$ and denominator $\xi_2(x)$.

We employ, (29) in this simulation as the controlled object. The fuzzy SMC is applied to the control system using the equation of control law in (10). Figure 4 shows signal tracking simulation, and Figure 4(a) shows the trajectory tracking between the desired signal and output signal using MATLAB/simulation. From this figure, the reference signal r_{ref} is marked with a red line color, and the output signals are marked with blue dotted line color. The simulation has different tracking between the desired and the output signal. Furthermore, the convergence of both the reference signal and the actual output is faster, and the performance of the fuzzy SMC control algorithm improves for the class of a SISO nonlinear system due to the error being close to zero. In addition, chattering errors in position tracking are reduced, and the control system achieves good performance.

Figure 4(b) illustrates the results of the magnification of Figure 4(a) at the state time of 7.6 to 7.8 seconds. It means that the trajectory tracking signal still has chattering, but it is very small. The magnitude of this catering error is different from the traditional SMC, which is quite significant if not the control system is not combined with the fuzzy logic system. The input u is exhibited in Figure 5. The signal of the trajectory control system varies with a certain amount that is different at any given time. Figure 6 shows the estimated $\hat{f}(\mathbf{x}, t)$ and estimated $\hat{g}(\mathbf{x}, t)$ signals to replace the quantities f and g in the control law. The estimated values of f and g are different and meet the control system's needs, so the adaptive fuzzy SMC control can be realized with minimal chattering error by approaching zero or zero value.

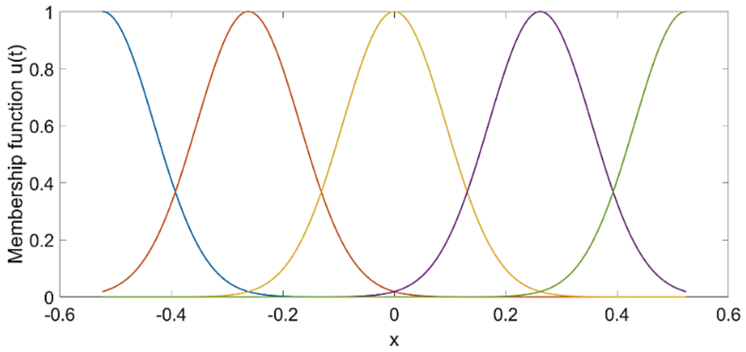
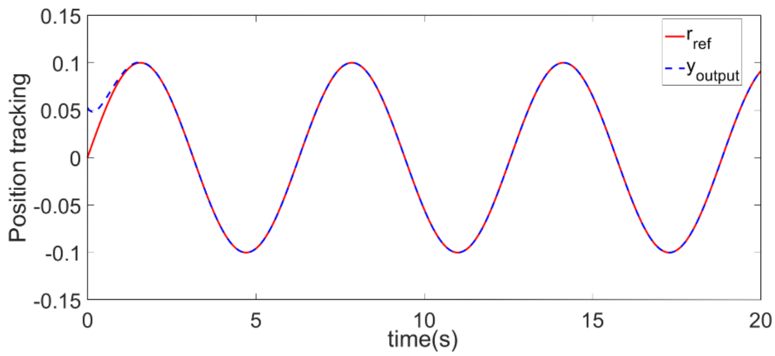
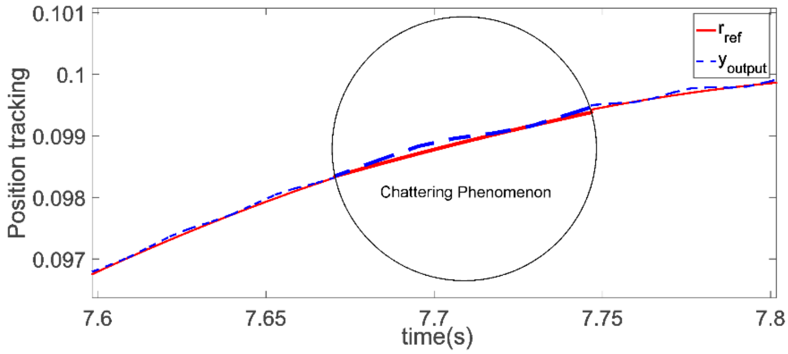


Figure 3. The membership function u(t) of adaptive fuzzy SMC



(a)



(b)

Figure 4. Signal tracking simulation of (a) sinusoidal position tracking and (b) magnification of chattering phenomenon

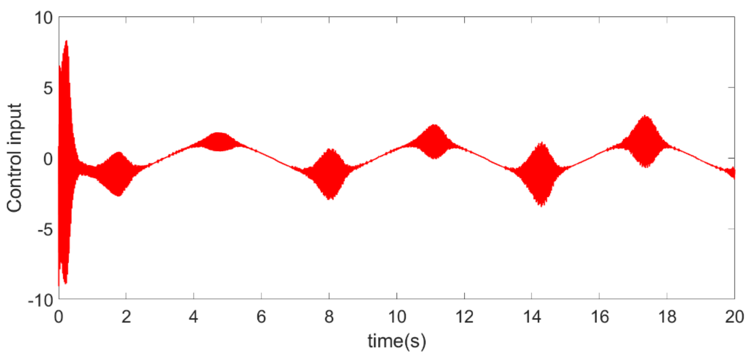


Figure 5. Control input u

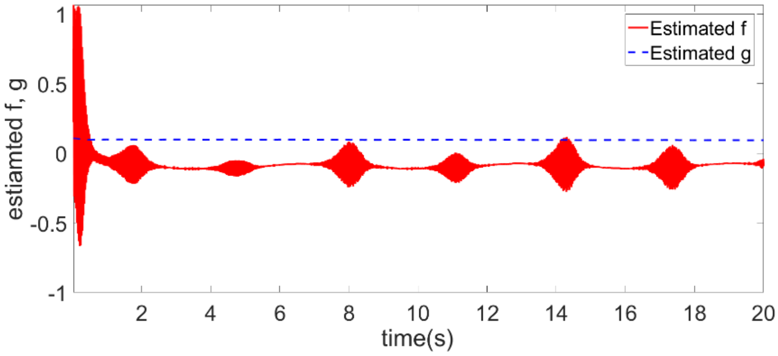


Figure 6. The estimated signal $\hat{f}(\mathbf{x},t)$ and $\hat{g}(\mathbf{x},t)$

Here is the comparison result between two different control algorithms, both using traditional SMC and adaptive fuzzy SMC. The traditional SMC method has a bigger error than the hybrid SMC control algorithm combined with the fuzzy system. By combining adaptive fuzzy and SMC, the chattering problem can be reduced to a minimum so that the resulting error is smaller or approaching zero value. A small error makes the control system on the inverted pendulum more robust from disturbances and performs better than the traditional SMC algorithm.

To evaluate the performance, we also compared the proposed control system with the previous studies in [24] with a similar controlled object. Table 1 shows the reference controller (MRC) model combined with fuzzy type-1 and type-2 has been investigated in a similar controlled plant. Based on the performance comparison, the proposed control algorithm using the AFSMC obtained the smallest ITSE of 0.0004587, or in other words, AFSMC had better control performance for the inverted pendulum.

Table 1. Comparison of a control algorithm for an inverted pendulum

Control Algorithms	ITSE
MRC [27]	0.00144
MRC+T1F with Tab. 5 [27]	0.0008619
MRC+T2F with Tab. 5 [27]	0.0008128
Proposed Algorithm (AFSMC)	0.0004587

4. CONCLUSION

Combining fuzzy logic systems and SMC algorithms is one of the methods for reducing chattering problems. We consider the merits of the fuzzy logic approach and SMC algorithm as well if they are combined. A single-stage inverted pendulum is employed in the system as the controlled object. MATLAB/Simulink simulated the hybrid control to evaluate this combination and achieved high-precision control of linear and nonlinear systems. The chattering problem on the trajectory signal of the inverted pendulum is eliminated using the combination of fuzzy logic system and SMC algorithm compared with the traditional SMC control system. The combination of the adaptive fuzzy and SMC algorithm demonstrates better control performance with fast response and robustness for industrial application.

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REFERENCES

[1] O. Apata and D. T. O. Oyedokun, "An overview of control techniques for wind turbine systems," *Sci. African*, vol. 10, p. e00566, Nov. 2020, doi: 10.1016/j.sciaf.2020.e00566.

[2] S. van Mourik *et al.*, "Introductory overview: Systems and control methods for operational management support in agricultural production systems," *Environ. Model. Softw.*, vol. 139, p. 105031, May 2021, doi: 10.1016/j.envsoft.2021.105031.

[3] J. P. Hespanha, D. Liberzon, and A. S. Morse, "Overcoming the limitations of adaptive control by means of logic-based switching," *Syst. Control Lett.*, vol. 49, no. 1, pp. 49–65, May 2003, doi: 10.1016/S0167-6911(02)00342-0.

[4] A. Salem, M. A. M. Hassan, and M. E. Ammar, “Tuning PID Controllers Using Artificial Intelligence Techniques Applied To DC-Motor and AVR System,” *Asian J. Eng. Technol.*, pp. 2321–2462, 2014.

[5] S. A. Touil, N. Boudjerda, A. Boubakir, and K. El Khamlichi Drissi, “A sliding mode control and artificial neural network based MPPT for a direct grid-connected photovoltaic source,” *Asian J. Control*, vol. 21, no. 4, pp. 1892–1905, 2019, doi: 10.1002/asjc.2007.

[6] J. Nowaková and M. Pokorný, “Intelligent controller design by the artificial intelligence methods,” *Sensors (Switzerland)*, vol. 20, no. 16, pp. 1–27, 2020, doi: 10.3390/s20164454.

[7] M. A. Khan, D. E. Z. Baig, B. Ashraf, H. Ali, J. Rashid, and J. Kim, “Dynamic modeling of a nonlinear two-wheeled robot using data-driven approach,” *Processes*, vol. 10, no. 3, 2022, doi: 10.3390/pr10030524.

[8] R. Solea and D. Cernega, “Sliding mode control for trajectory tracking problem-performance evaluation,” 2009, pp. 865–874.

[9] J. Hu, H. Zhang, H. Liu, and X. Yu, “A survey on sliding mode control for networked control systems,” *Int. J. Syst. Sci.*, vol. 52, no. 6, pp. 1129–1147, Apr. 2021, doi: 10.1080/00207721.2021.1885082.

[10] S. J. Gambhire, D. R. Kishore, P. S. Londhe, and S. N. Pawar, “Review of sliding mode based control techniques for control system applications,” *Int. J. Dyn. Control*, vol. 9, no. 1, pp. 363–378, Mar. 2021, doi: 10.1007/s40435-020-00638-7.

[11] X. Yu, Y. Feng, and Z. Man, “Terminal sliding mode control – an overview,” *IEEE Open J. Ind. Electron. Soc.*, vol. 2, pp. 36–52, 2021, doi: 10.1109/OJIES.2020.3040412.

[12] D. H. Vu, S. Huang, and T. D. Tran, “Hierarchical robust fuzzy sliding mode control for a class of SIMO under-actuated systems with mismatched uncertainties,” *Telkomnika (Telecommunication Comput. Electron. Control.*, vol. 17, no. 6, pp. 3027–3043, 2019, doi: 10.12928/TELKOMNIKA.v17i6.13176.

[13] V. Utkin and H. Lee, “Chattering problem in sliding mode control systems,” *IFAC Proc. Vol.*, vol. 39, no. 5, p. 1, 2006, doi: 10.3182/20060607-3-IT-3902.00003.

[14] N. Cibiraj and M. Varatharajan, “Chattering reduction in sliding mode control of quadcopters using neural networks,” *Energy Procedia*, vol. 117, pp. 885–892, Jun. 2017, doi: 10.1016/j.egypro.2017.05.207.

[15] B. Li, J. Zhu, R. Zhou, and G. Wen, “Adaptive neural network sliding mode control for a class of SISO nonlinear systems,” *Mathematics*, vol. 10, no. 7, p. 1182, Apr. 2022, doi: 10.3390/math10071182.

[16] S. C. Yogi, V. K. Tripathi, and L. Behera, “Adaptive Integral Sliding Mode Control Using Fully Connected Recurrent Neural Network for Position and Attitude Control of Quadrotor,” *IEEE Trans. Neural Networks Learn. Syst.*, vol. 32, no. 12, pp. 5595–5609, Dec. 2021, doi: 10.1109/TNNLS.2021.3071020.

[17] X. Cheng, H. Liu, and W. Lu, “Chattering-suppressed sliding mode control for flexible-joint robot manipulators,” *Actuators*, vol. 10, no. 11, 2021, doi: 10.3390/act10110288.

[18] R. Khalili Amirabadi, O. S. Fard, and A. Mansoori, “A novel fuzzy sliding mode control approach for chaotic systems,” *Iran. J. Fuzzy Syst.*, vol. 18, no. 6, pp. 133–150, 2021, doi: 10.22111/ijfs.2021.6338.

[19] K. A. Baghaei, H. Ghaffarzadeh, S. A. Hadigheh, and D. Dias-da-Costa, “Chattering-free sliding mode control with a fuzzy model for structural applications,” *Struct. Eng. Mech.*, vol. 69, no. 3, pp. 307–315, 2019, doi: 10.12989/sem.2019.69.3.307.

[20] M. S. Ahmed, A. Mary, and H. H. Jasim, “Model and chattering free adaptive fuzzy SMC for robotic manipulator systems,” *Texas J. Eng. Technol.*, pp. 33–43, 2022.

[21] N. Nafia, A. El Kari, H. Ayad, and M. Mjahed, “A robust type-2 fuzzy sliding mode controller for disturbed MIMO nonlinear systems with unknown dynamics,” *Automatika*, vol. 59, no. 2, pp. 194–207, 2018, doi: 10.1080/00051144.2018.1521568.

[22] M. Khazaee, A. H. D. Markazi, and E. Omidi, “Adaptive fuzzy predictive sliding control of uncertain nonlinear systems with bound-known input delay,” *ISA Trans.*, vol. 59, pp. 314–324, 2015, doi: 10.1016/j.isatra.2015.10.010.

[23] C. Zhao and L. Guo, “Control of Nonlinear Uncertain Systems by Extended PID,” *IEEE Trans. Automat. Contr.*, vol. 66, no. 8, pp. 3840–3847, 2021, doi: 10.1109/TAC.2020.3030876.

[24] E. H. Karam, N. A. Al-Awad, and N. S. Abdul-Jaleel, “Design nonlinear model reference with fuzzy controller for nonlinear SISO second order systems,” *Int. J. Electr. Comput. Eng.*, vol. 9, no. 4, p. 2491, 2019, doi: 10.11591/ijece.v9i4.pp2491-2502.

[25] J. J. Wang and T. Kumbasar, “Optimal PID control of spatial inverted pendulum with big bang-big crunch optimization,” *IEEE/CAA J. Autom. Sin.*, vol. 7, no. 3, pp. 822–832, 2020, doi: 10.1109/JAS.2018.7511267.

[26] K. Zhou, “Study on application of fuzzy control model in phosphate removal system of sewage water,” *Appl. Water Sci.*, vol. 12, no. 3, 2022, doi: 10.1007/s13201-021-01502-5.

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