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The production-inventory model with imperfect, rework, and scrap items under stochastic demand

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ABSTRACT

Inventory is crucial in maintaining a smooth production process and meeting consumer demand for manufacturing companies. This research focuses on production problems involving defects, rework, and scrap items in stochastic demand. This research aims to develop a production-availability model by minimizing the expectation of total cost (ETC). The model includes four main decision variables, namely production quantity (Q), safety factor (k), production rate (P), and rework rate (P1). This research uses the Aquila optimizer algorithm to optimize the objective function. It compares with the heuristic procedure and Harris Hawk optimization algorithm. The results showed that the Aquila optimizer algorithm successfully optimized the production-availability problem. A comparison between algorithms indicates that the Aquila optimizer algorithm performs equivalently to the Harris Hawk optimization algorithm and outperforms the heuristic procedure. Sensitivity analysis shows that increasing demand uncertainty increases ETC and k. At the same time, it can decrease Q.

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1. INTRODUCTION

In the era of globalization and increasingly fierce business competition, supply chain management has become a crucial element in company success [1]. The supply chain includes raw material procurement, production planning, and inventory management [2], [3]. While changing market dynamics, companies must develop effective strategies to manage their supply chains. One concept that becomes the main focus is the close relationship between the supply chain and the production-inventory model. The supply chain supports the company's smooth operation, from procuring raw materials to distributing final products to customers [4]–[6]. With uncertainty in market demand, good supply chain management can help companies respond to changes more quickly and efficiently. On the other hand, the production-inventory model is the foundation for planning and managing production and inventory [7]. A harmonious relationship between the supply chain and the production-inventory model is the key for companies to minimize costs, increase efficiency, and optimally meet customer demand. Accurate and comprehensive models must be developed to manage inventory and production in production and supply chain management. In today's production systems, one of the biggest obstacles is managing stochastic demand uncertainty, which causes demand to fluctuate at random [8].

The production-inventory model becomes essential in optimizing the production process and stock management. However, in practice, many factors can affect the efficiency and effectiveness of the model. One of the main challenges faced by companies is the presence of imperfections in production goods, rework processes, and the occurrence of scrap goods. Imperfections can occur for various reasons, such as production defects or errors in the manufacturing process [9], [10]. The rework process is needed to repair goods that do not meet quality standards so that they can be reused or sold [11]–[13]. However, this process also requires additional costs and time that cannot be ignored. In addition, scrap is also a problem that needs to be handled properly, as it can result in financial losses for the company [11], [14]. In addition, stochastic or uncertain demand is also one of the challenges in inventory management [10]. Fluctuating and unpredictable demand give companies an active and responsive strategy to maintain adequate availability of goods without causing unwanted inventory surpluses. Therefore, there is a need to develop a production-inventory model that considers imperfections, rework, and scrap, along with stochastic demand. Considering all these factors, the model is expected to assist companies in optimizing inventory management decisions to achieve efficiency by minimizing total costs.

Past research on production-availability models involving defective products, rework, and waste has attracted the attention of several researchers who contribute to expanding the understanding of the complexity of this problem. One of the main contributors is Hejazi *et al*. [15], who proposed a model that requires rework on defective products to avoid waste. The study found that by integrating the rework process into the model, companies can reduce the amount of waste and improve their operational efficiency. In addition, research by Sarkar [16], Lin and Su [17], Paul *et al*. [18], Manna *et al*. [19], Chang and Ho [20], and Manna *et al*. [21] also contributed by incorporating defective products into their production-availability model. Their results show that an inventory management strategy that considers defective products can help companies optimize the use of their resources. On the other hand, research by Sanjai and Periyasamy [22], Öztürk [23], and Khara *et al*. [24] involving defective products and rework processes highlights the importance of efficiency in inventory management to avoid losses. Meanwhile, studies by Su *et al*. [10] and Krishnamoorthi and Panayappan [13] have enriched the understanding of models that consider defective products and waste in their production-availability models. They found that by integrating waste processing into the model, companies can reduce production costs and improve environmental sustainability. However, only studies by Su *et al*. [10] and Sarkar *et al*. [25] have successfully incorporated stochastic elements in demand modelling, showing its importance in dealing with uncertainty in demand.

Although various models have been proposed, a comprehensive production-availability model considers defective products, rework, and scrap and faces stochastic challenges in demand. Previous research has provided valuable insights into integrating these elements into the model. However, there still needs to be gaps in our understanding of how these factors interact and impact overall inventory management decisions. The lack of models that can accommodate all these factors simultaneously implies that decision-makers do not yet have the optimal tools to cope with the complexities in a constantly changing production and demand environment. Therefore, there is a need for a model that can integrate all aspects, such as defective products, rework, and scrap, and deal with stochastic challenges so that decision-makers make more informed and effective decisions in managing their inventory, improve operational efficiency and respond better to changes in the market. This research aims to develop a comprehensive production-inventory model that considers defective products, rework, and scrap and faces stochastic challenges in demand. In addition, this research aims to offer the metaheuristic procedure of the Aquila algorithm as a method to find the optimal decision variables [26]. The Aquila optimizer, which is inspired by the hunting behavior of the Aquila Hawk that can hunt prey, has proven to be effective in solving various problems, including industrial engineering optimization problems [27], image classification [28], and population forecast [29]. We seek to apply this procedure to solve complex problems in production-availability models based on its effectiveness in various contexts. Thus, this research not only aims to fill the gap in understanding comprehensive production-availability models but also to test and validate the potential of the Aquila algorithm as an effective optimization tool.

This research contributes significantly by providing new insights into a comprehensive production-availability model, which considers defective products, rework, and scrap and faces stochastic challenges in demand. Such contributions include not only the development of a new model that is expected to help companies improve production efficiency and optimize inventory levels but also provide a solid foundation for adaptive decision-making amid unpredictable market dynamics in this era of uncertainty. In addition, this research proposes a new procedure for the Aquila algorithm to optimize the production-inventory model problem. With this new procedure, this research enriches the optimization procedures used in production-availability model optimization, making valuable contributions to developing analytical methods in the context of complex inventory management. Thus, this research is expected to provide practical guidance for companies in managing their inventories more effectively and enrich the academic literature with new contributions in this field.

2. DESCRIPTION PROBLEM AND PROPOSED MODEL

2.1. System characteristics

Inventory systems in manufacturing companies have complex characteristics and involve various types of significant costs in the production process. Some of the essential costs involved in this system include setup cost (K), regular production cost (c), scrap product disposal cost (cs), rework cost (cr), and finished product holding cost (h). Each cost element is essential in optimizing production efficiency and inventory management. However, a significant challenge in inventory management is the fluctuation or stochasticity of consumer demand (D). Erratic demand every month creates the need to implement a safety strategy. Therefore, companies must identify and determine the number of safety products (S) to maintain product availability when demand suddenly increases or significant fluctuations occur.

In addition, companies should also consider production aspects such as production quantity (Q), production rate (P), and rework rate (P1) in their inventory system. The P should be set to meet or even exceed D, ensuring adequate product availability. Companies must also consider the P1 to address defective or non-standard products. Managing this rework rate is critical in minimizing losses and ensuring the products' quality meets the set standards.

2.2. Assumptions and notations

This section presents the assumptions and notations of the proposed model. Some of the key assumptions used as a foundation in designing the model are i) ordering costs are constant, meaning that the costs involved in the ordering process do not depend on the amount of ordering; ii) there is no shortage of raw materials, so inventory is always available to meet demand; iii) stating that storage capacity is unlimited, thus removing the physical limitations associated with the amount of goods that can be stored; iv) demand levels are stochastic and normally distributed, thus allowing the model to account for demand variations statistically; and v) stating that regular storage costs and rework costs are assumed to be equal, thus simplifying the overall cost analysis. These five assumptions form the theoretical basis for developing inventory models in the context of production availability problems. The notations used in this research are:

P	:	production rate per unit of time
P_1	:	rework rate per unit of time
Q	:	production lot size per cycle
D	:	number of demands per unit of time
d	:	production rate of defective items per unit of time
d_1	:	production rate of leftover products during rework per unit time
d_2	:	production rate of imperfect quality items during rework per unit time
d^0	:	total production rate of leftover products and imperfect quality items per unit of time
t_1	:	production uptime in the presence of inventory
t_2	:	time spent reworking
t_3	:	time used when inventory runs out
T	:	cycle length, $T = t_1 + t_2 + t_3$
q	:	the proportion of defective items produced
θ	:	the proportion of scrap in defective items
γ	:	the proportion of imperfect quality items in defective items
β	:	the proportion of reprocessing in defective items
θ_1	:	the proportion of scrap generated during rework
γ_1	:	the proportion of imperfect quality items produced during rework
$E(.)$	:	estimated value
K	:	setup cost for each production (IDR)
H	:	the maximum level of on-hand inventory of perfect items in units when the rework process ends
c	:	production cost per unit (IDR)
H_1	:	the maximum level of on-hand inventory of perfect items in units when the regular production process stops
c_S	:	disposal cost per unit of leftover product (IDR)
h	:	inventory cost per unit (IDR)
c_R	:	rework cost per rework product (IDR)
σ	:	standard deviation of demand per unit of time
k	:	safety factor
π	:	lost sale cost (IDR)
$fs(k)$	:	probability density function of the normal distribution
$Fs(k)$	:	cumulative distribution function of the normal distribution

2.3. Proposed production-inventory model

This study proposes an inventory production model involving imperfect, reworked, and scrap items under stochastic demand. The proposed mathematical model aims to minimize the total inventory cost. Figure 1 shows the proposed model's available inventory and allowable order levels. The time duration of a cycle is the sum of the regular production time (t_1), rework time (t_2), and production downtime (t_3). The t_1, t_2, and t_3 are presented in (1)-(3), respectively. Meanwhile, based on Figure 1, the maximum level of on-hand inventory of perfect items in units when the regular production process stops (H_1) and the maximum level of on-hand inventory of perfect items in units when the rework process ends (H) can be formulated in (4) and (5). In the proposed model, products are also defective during production, so the inventory level needs to be estimated. An illustration of the on-hand inventory level of defective items is shown in Figure 2. Defective items produced during t_1 are calculated as in (6).

$$t_1 = \frac{Q}{P'} = \frac{H_1}{P-d-D} \quad (1)$$

$$t_2 = \frac{H-H_1}{P_1-d^0-D} \quad (2)$$

$$t_3 = \frac{H}{D} \quad (3)$$

$$H_1 = (P-d-D)t_1 \quad (4)$$

$$H = H_1 + (P_1 - d^0 - D)t_2 \quad (5)$$

$$G = d.t_1 = qQ \quad (6)$$

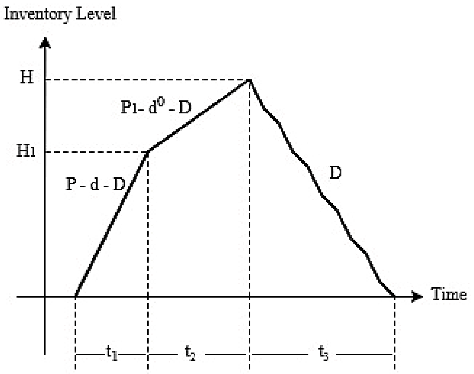

Figure 1. Illustration of perfect inventory on hand

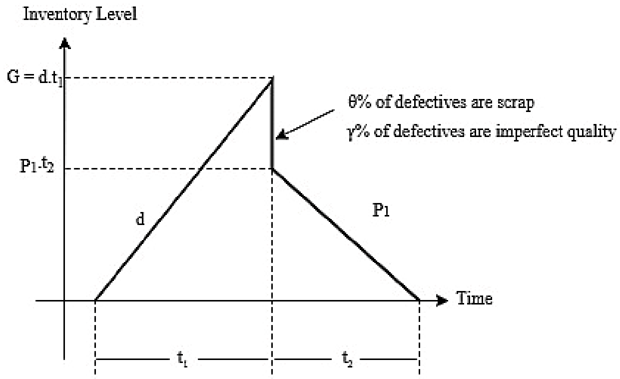

Figure 2. Illustration of inventory levels of a defective item

In this proposed model, the main focus is to elaborate the model based on the illustrated on-hand inventory of reworkable items, imperfect quality items, and scrap items, as shown in Figures 3-5. Figure 3 shows the illustrated inventory levels of reworkable items, while Figures 4 and 5 display the inventory levels of imperfect quality items and scrap items. In this context, it is assumed that the proportion β of defective items can be regarded as reworkable items, the proportion θ as scrap items, and the proportion γ as imperfect quality items. The proposed model for the number of reworkable items, items of imperfect quality, and scrap items produced during a regular production process can be formulated using (7)-(9).

$$G_1 = d\beta.t_1 = P_1 t_2 = \beta qQ \quad (7)$$

$$G_2 = d\gamma.t_1 = \gamma qQ \quad (8)$$

$$G_4 = d\theta.t_1 = \theta qQ \quad (9)$$

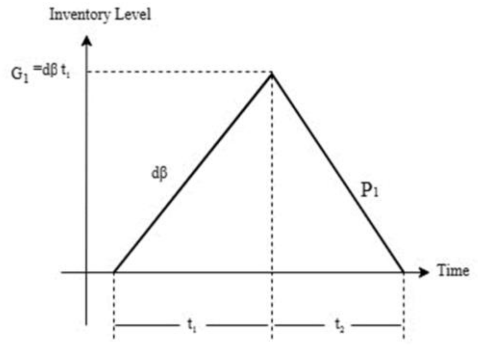

Figure 3. On-hand inventory level of reworkable items

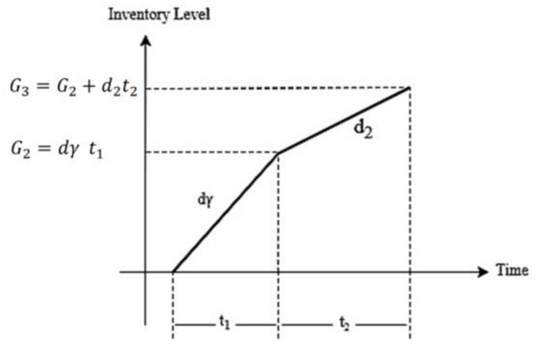

Figure 4. On-hand inventory level of imperfect quality items

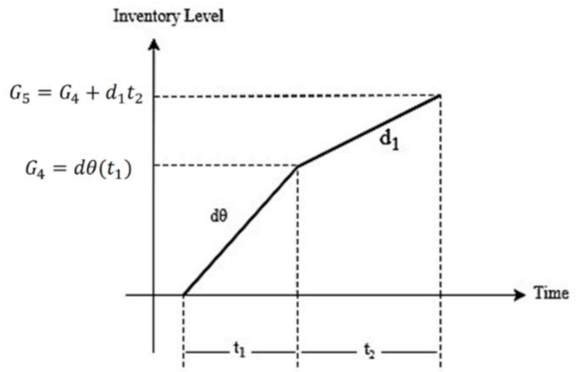

Figure 5. On-hand inventory level of scrap items

During the rework process, the scrap and imperfect quality production rates are crucial parameters to consider. These two parameters are represented by (10) and (11). To measure the total imperfect quality goods and scrap goods produced during this process, equations (12) and (13) are used. Furthermore, in this context, it is assumed that d^0 is the sum of the production rates of scrap and imperfect quality goods produced during the rework process. This measurement is expressed in (14). Furthermore, the inventory levels in the hands of H_1 and H can be estimated based on (15) and (16). Where, $P - d - D = P\left(1 - q - \frac{D}{P}\right) = PA_1$ and $P_1 - d^0 - D = P_1\left(1 - \theta_1 - \gamma_1 - \frac{D}{P_1}\right) = P_1A_2$. Hence, the value of T is calculated based on (17).

$$d_1 = P_1\theta_1 \quad (10)$$

$$d_2 = P_1\gamma_1 \quad (11)$$

$$G_3 = G_2 + d_2t_2 = (\gamma + \gamma_1\beta)qQ \quad (12)$$

$$G_5 = G_4 + d_1t_2 = (\theta + \theta_1\beta)qQ \quad (13)$$

$$d^0 = d_1 + d_2 = P_1(\theta_1 + \gamma_1) \quad (14)$$

$$H_1 = (P - d - D)t_1 = A_1Q \quad (15)$$

$$H = H_1 + (P_1 - d^0 - D)\frac{\beta qQ}{P_1} = A_1Q - \beta + A_2\beta qQ = (A_1 + A_2\beta q)Q \quad (16)$$

$$T = t_1 + t_2 + t_3 = \frac{[1 - q(1 - \beta(1 - \theta_1 - \gamma_1))Q]}{D} \quad (17)$$

Since product demand is stochastic with a normal distribution, decision-makers must increase inventory levels by involving safety stock (S). The S of the product is modeled in (18), which is adapted from the model proposed by Utama *et al*. [30]. In addition, the expected demand shortage of the period $Q\ /\ D = T$ is modeled in (19), where $\Psi_{(k)} = \{fs(k) - k[1 - Fs(k)]\}$.

$$S = k\sigma\sqrt{\frac{Q}{D}} \tag{18}$$

$$\sigma\sqrt{\frac{Q}{D}}\Psi_{(k)} \tag{19}$$

The expectation of total inventory cost in each cycle is modeled in (20). Meanwhile, the proportion of defective products is a random variable, so the production cycle length T is also a random variable. Then, the expected cycle length $E(T)$ can be formulated in (21).

$$TC(Q,S) = K + cQ + c_s\theta qQ + c_s\theta_1\beta cQ + c_R\beta qQ + h\left(\frac{H_1(t_1)}{2} + \frac{(H_1+H)(t_2)}{2} + \frac{H(t_3)}{2} + \frac{G(t_1)}{2} + \frac{G_1(t_2)}{2} + k\sigma\sqrt{\frac{Q}{D}}\right) + \pi\left(\sigma\sqrt{\frac{Q}{D}}\Psi_{(k)}\right) = K + cQ + c_s(\theta + \theta_1\beta)qQ + c_R\theta cQ + Q^2\left(h\left(\frac{A_1}{2P} + \frac{(A_1+A_2\beta q)\beta q}{2P_1} + \frac{(A_1+A_2\beta q)^2}{2D}\right) + \frac{q}{2P} + \frac{\beta^2q^2}{2P_1} + k\sigma\sqrt{\frac{Q}{D}}\right) + \pi\left(\sigma\sqrt{\frac{Q}{D}}\Psi_{(k)}\right) \tag{20}$$

$$E(T) = \frac{[1-E(q)(1-\beta(1-\theta_1-\gamma_1))]Q}{D} = \frac{(1-E_1)Q}{D} \tag{21}$$

Where, $E_1 = E_{(q)}(1 - \beta(1 - \theta_1 - \gamma_1))$, $E_2 = 1 - E_{(q)} - \frac{D}{P}$, $E_3 = E_{\left(\frac{q}{1-q-\frac{D}{P}}\right)}$.

Based on the (1)-(19), the total expected inventory cost in a one-time horizon can be modeled in (22). Minimize the total cost of the inventory system, can be achieved by simultaneously determining the optimal decision variables, namely the production lot size per cycle (Q), safety factor (k), production rate (P), and rework production rate (P_1). The product k in a normal distribution must be greater than 0 and cannot exceed 2.99. Meanwhile, the P and P_1 must be higher than the D.

$$E(TCU(Q,k,P,P_1)) = \frac{E(TC(Q,S))}{E(T)} = \frac{KD}{(1-E_1)Q} + \frac{(c + E_{(q)}[c_s(\theta+\theta_1\beta) + c_R\beta])D}{(1-E_1)} + \frac{hQD}{2P_1(1-E_1)}\left(\left(\left(1 + \frac{P_1A_2}{D}\right)A_2 + \frac{1}{h}\right)\beta^2E_{(q^2)} + 2\left(1 + \frac{P_1A_2}{2}\right)\beta E_{(q)}E_2 + \left(1 + \frac{PE_2}{D} + E_3\right)\frac{P_1E_2}{P} + k\sigma\sqrt{\frac{(1-E_1)Q}{D}}\right) + \frac{D}{(1-E_1)Q}\,\sigma\pi\sqrt{\frac{(1-E_1)Q}{D}}\Psi_{(k)} \tag{22}$$

3. METHODS

3.1. Proposed method Aquila optimizer algorithm

To optimize the production-inventory model problem with imperfect, reworked, and scrap items under stochastic demand, this study proposes the advanced procedure of Aquila optimizer. The Aquila optimizer is an algorithm inspired by the hunting behavior of Aquila Hawks that can hunt prey [28]. The Aquila optimizer algorithm pseudocode is presented in Algorithm 1.

The first phase of Aquila in hunting prey is to expand the search. Where the Aquila is at a height to determine the location of its prey, this behavior is described by (23).

$$X_1(t+1) = X_{best}(t) \times \left(1 - \frac{t}{T}\right) + (X_M(t) - X_{best}(t) * rand) \tag{23}$$

Where, $X_1(t+1)$ is the result for t in the next iteration. $X_{best}(t)$, the best result obtained by iteration t (prey position). The number of iterations $(1-t/T)$ is used to expand the search. $X_M(t)$ indicates the average value of iteration t calculated based on $X_M(t)=\frac{1}{N}\sum_{t=1}^{N}X_i(t)$. Rand is a random value between 0 and 1.

The second phase is when the prey has been found at a height, and the Aquila starts to attack (X_2). In this phase, the Aquila circles above its prey and prepares the terrain before attacking after finding the prey area from a height. This method's behavior is contour flying with a short gliding attack. This behavior is called narrow exploration, modeled in (24). D is the dimensional space, and $Levy(D)$ denotes the distribution function of the step size. Meanwhile, $X_R(t)$ is a random solution from the range $[1\ N]$ at iteration i.

$$X_2(t+1)=X_{best}(t)\times Levy(D)+X_R(t)-(y-x)*rand \tag{24}$$

In the third phase (X_3), the Aquila captures prey by flying low and vertically with a slow attack. This phase is referred to as expanded exploitation. This behavior is modeled in (25). Where, $X_3\ (t+1)$ is the result for t in the next iteration. $X_{best}(t)$ indicates the best result by iteration t (prey position). $X_M(t)$ indicates the average value of iteration t. Rand is a random value between 0 and 1. α and δ are parameters for exploitation adjustment (0.1).

$$X_3(t+1)=\left(X_{best}(t)\times X_M(t)\right)\times\alpha-rand+\left((UB-LB)\times rand+LB\right)\times\delta \tag{25}$$

The fourth phase describes walking and catching prey in the final position. This phase is referred to as narrowed exploitation. This behavior is modeled in (26). $X_4(t+1)$ is the result of t in the next iteration (X_3). QF is the quality function for the balance search strategy. $X_{best}(t)$, the best result by iteration t (prey position). G_1 indicates the variation of Aquila movement to track the prey. $QF(t)$ is the quality function for iteration t. The rand value comes from a random value of 0 to 1. t and T are the maximum number of iterations, respectively.

$$X_4(t+1)=QF\times X_{best}(t)-(G_1\times X(t)\times rand)-G_2\times Levy(D)+rand\times G_1 \tag{26}$$

Algorithm 1 Pseudocode Aquila optimizer

```
Initialization phase:
Initialize the population X and parameters.
While (The end condition is not met) do
  Calculate the fitness function values.
  X_best(t) = Determine the best-obtained solution according to the fitness values.
  for (i=1,2.,N) do
   Update the mean value of the current solution X_M(t).
   Update the x,y,G_1,G_2,Levy(D).
   if t ≤ (2/3)*T then
    if rand ≤ 0.5 then
     ▷ Step 1: Expanded exploration (X_1)
     Update the current solution using the Equation (23)
     if Fitness (X_1(t+1)) < Fitness (X(t)) then X(t) = (X_1(t+1))
      if Fitness (X_1(t+1)) < Fitness (X_best(t)) then X_best(t) = X_1(t+1)
      end if
     end if
    else
     ▷ Step 2: Narrowed exploration (X_2)
     Update the current solution using the Equation (24)
     if Fitness (X_2(t+1)) < Fitness (X(t)) then X(t) = (X_2(t+1))
      if Fitness (X_2(t+1)) < Fitness (X_best(t)) then X_best(t) = X_2(t+1)
      end if
     end if
    else
    if rand ≤ 0.5 then
     ▷ Step 3: Narrowed exploration (X_3)
     Update the current solution using the Equation (25)
     if Fitness (X_3(t+1)) < Fitness (X(t)) then X(t) = (X_3(t+1))
      if Fitness (X_3(t+1)) < Fitness (X_best(t)) then X_best(t) = X_3(t+1)
      end if
     end if
    else
     ▷ Step 4: Narrowed exploration (X_4)
     Update the current solution using the Equation (26)
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```
          if Fitness (X_4(t+1)) < Fitness (X(t)) then X(t) = (X_4(t+1))
            if Fitness (X_4(t+1)) < Fitness (X_best(t)) then X_best(t) = X_4(t+1)
            end if
          end if
        end if
      end if
    end for
  end while
  Return the best solution (X_best)
```

3.2. Data and experimental procedures

The data for this research comes from a case study of a manufacturing industry that produces plastic packaging in Indonesia. Various parameters relevant to the inventory problem, such as D=1,6079 kg, P=1,7000 kg, P_1=5,000 kg, q=0.05%, β=1%, θ_1=0%, γ_1=0%, θ=0%, σ=4,541 kg, c=27,557.34 Rp/kg, c_R=30,644.57 Rp/kg, c_s=11,000 Rp/kg, h=IDR 25, K=IDR 1,162,858.77, π=IDR 45,000. This research utilizes Aquila optimizer as an optimization tool with the primary objective of minimizing total cost-efficiently. The decision variables that are the focus of exploration in this study involve Q, k, P, and P1. Table 1 presents the Aquila algorithm parameters used to optimize the inventory problem.

Testing the effectiveness of the Aquila optimizer is done by comparing it with the heuristic algorithm and Harris Hawk optimization algorithm. In addition, this study also evaluates the effect of demand fluctuations (D) by conducting a sensitivity analysis of changes in the standard deviation (σ) of demand on the objective function and decision variables. All inventory optimization experiments were conducted using MATLAB R2018a platform on an MSI Modern 14 C5M-005 Ryzen 5 5625U computer with 8 GB RAM and 512 GB storage, operating under Windows 11 system.

Table 1. Parameter data of the Aquila algorithms for the inventory optimization problem

Parameter	Value
Populations number	1000
Maximum iteration	1000
Number of decision variable	4
Upper bound	[16079, 2.99, 18000, 7000]
Lower bound	[1, 0, 17000, 5000]

4. RESULTS AND DISCUSSION

4.1. Optimization with Aquila optimizer

This study successfully optimized the production-availability model by considering imperfections, reworked goods, and scrap goods under stochastic stock demand conditions. The optimization process is performed using the Aquila optimizer algorithm method. The result of this optimization shows that the total cost generated is IDR 536,308,681.20. The optimal decision variables identified involved the Q of 554, k of 2.24, P of 18000, and P1 of 7000. These findings reflect the efficiency and effectiveness of the proposed model in dealing with inventory challenges with uncertainty, rework, and scrap. In addition, these results can serve as guidelines for practitioners and decision-makers in managing inventory more efficiently and optimizing costs significantly.

4.2. Algorithm comparison

A comparison of algorithms for optimization of production-inventory problems can be seen in Figure 6. The optimization results using the Aquila optimizer algorithm and Harris Hawk optimization resulted in an estimated total cost (ETC) of IDR 536,308,681.20. In comparison, the heuristic procedure resulted in an ETC of IDR 555,256,012.11. The algorithm comparison analysis shows that the Aquila optimizer algorithm produces a solution equivalent to the Harris Hawk optimization algorithm and superior to the heuristic procedure. The proposed method, the Aquila optimizer algorithm, reduced the expected cost by 3.22%. This result indicates that the procedure has effective exploration capabilities [28]. Furthermore, it was found that Aquila's behaviors, such as expanded and narrowed exploration, proved effective in exploring the solution space of production-availability problems. Thus, this study confirms that the Aquila optimizer algorithm can be an excellent choice to tackle complex problems in inventory management, especially with its ability to reduce costs significantly.

The findings of this study have some significant consequences. The results of the algorithm comparison analysis show that the Aquila optimizer algorithm is equivalent to the Harris Hawk optimization algorithm and superior to heuristic procedures. It confirms that the Aquila optimizer algorithm has excellent potential as an effective tool in solving complex problems in inventory management. This algorithm allows companies to obtain optimal solutions in their inventory management. In addition, this study also provides insight into the possibility of improving efficiency in inventory management in the future. With the confirmation that the Aquila optimizer algorithm can produce good solutions, this research provides the impetus for the further development and application of the algorithm in a broader context. This means that in the future, the Aquila optimizer algorithm will be an invaluable tool for companies to reduce their costs and improve their operational efficiency significantly.

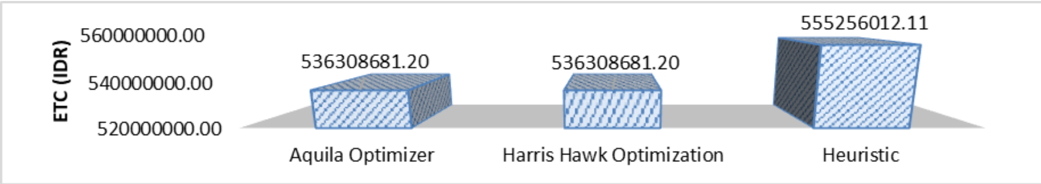


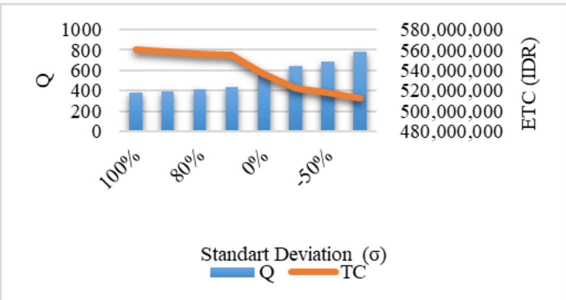
Figure 6 Comparison of algorithms for optimization of production-inventory problems

4.3. Sensitivity analysis of changes in standard deviation (σ)

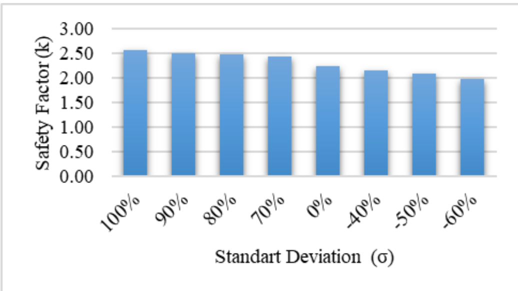
In the sensitivity analysis to changes in σ, Figure 7 illustrates the impact of changes in σ. Figure 7(a) influences the effect of changes in σ on the expectation of ETC and Q. Moreover, Figure 7(b) influences the impact of changes in σ on k. The results show that the higher the σ value, the more volatile the product demand. The main finding is that an increase in the σ value leads to a decrease in the Q with an increase in the k value. It indicates that the higher the demand uncertainty, the smaller the planned production to avoid the risk of losing sales [30]. Although the P and P1 are not affected by changes in the σ, this result shows that they maintain a fixed value, probably due to other factors unaffected by demand fluctuations.

The increase in the k results from efforts to avoid the risk of losing sales. However, at the same time, it also increases the shelf cost, increasing the expectation of ETC [30]. Therefore, inventory management must consider the trade-off between the risk of losing sales and storing costs to optimize the total inventory cost. A deeper understanding of the impact of the sensitivity of σ changes on some inventory management provides valuable insights for practitioners and researchers in developing more effective strategies for managing demand uncertainty and improving supply chain performance.

This finding suggests that the more uncertain the demand, the smaller the planned production to avoid the risk of losing sales. Consequently, companies tend to reduce the risk of lost sales by reducing the planned production quantity. However, increasing the k to avoid the risk of lost sales does not come without consequences. Such efforts increase shelf costs due to more extensive inventories, increasing the ETC. Therefore, management should carefully consider balancing managing the risk of lost sales and minimizing inventory costs to achieve optimal results. From these findings, there are opportunities for further research that focus on developing inventory management strategies that are more adaptive and responsive to variability in demand.



(a)



(b)

Figure 7. Sensitivity analysis of σ changes to (a) Q, ETC, and (b) k

5. CONCLUSION

Based on this research, the production-inventory model that considers defective items, reprocessing, and leftover items on stochastic stock demand significantly impacts total costs. The results show that the Aquila optimizer algorithm optimizes the production-inventory problem with the same effectiveness as the Harris Hawk optimization algorithm. Furthermore, a comparison between the algorithms shows that the Aquila optimizer algorithm not only produces solutions similar to the Harris Hawk optimization algorithm but can also provide better solutions than heuristic procedures. Sensitivity analysis found increased uncertainty in demand increased total cost and safety factors. However, interestingly, the Q decreased as the uncertainty in demand increased. It provides valuable insights in managing inventory by considering the uncertainty in the demand factor. Thus, this research develops a complex production-inventory model and contributes to understanding how uncertainty can affect inventory management decisions.

Although this research successfully developed a production-inventory model considering defective items, remanufacturing, and waste in the context of stochastic demand, some limitations still need to be noted. First, the model only considers four main decision variables, namely Q, k, P, and P1. The possibility of additional variables affecting inventory decisions could be an exciting research area to explore further. In addition, although the Aquila optimizer algorithm successfully minimizes total cost and provides comparative results with the Harris Hawk optimization algorithm, further tests are still needed regarding the effectiveness of this algorithm in more complex production-availability scenarios. In addition, further research is needed to evaluate the model's performance in practical and dynamic situations that can occur in everyday production environments. Therefore, as a future research direction, it is necessary to explore further the development of a more comprehensive model and further testing and validation of the optimization methods used to improve the applicability and generalizability of the findings of this research.

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