Financing model for demand response information services with bundled incentives

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ABSTRACT

This study attempts to build a new model for information service financing schemes by considering utility functions to measure heterogeneous consumer satisfaction. This model was developed by involving a combination of reverse charging, demand response, and heterogeneous incentive models, and considering the quality of user service measured by a quasi-linear utility function against the information service financing scheme. The incentive financing scheme is applied to a local data server, including traffic during peak hours and off-peak hours. This internet incentive financing model is solved using the LINGO 13.0 application. Furthermore, the development model for incentive financing for information services based on demand response and bundling in the information service financing scheme is subjected to sensitivity analysis with the aim of identifying parameters that affect model performance. Based on the analysis that has been done, the results of this study indicate that the new model in the incentive financing scheme for information services with a quasi-linear utility function involving a combination of reverse charging, demand response, and heterogeneous incentive models produces an optimal solution in a fixed cost financing scheme for data traffic usage during peak hours and off-peak hours.

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1. INTRODUCTION

In today's era, the development of information technology, especially the internet, has made the internet a basic need for modern society [1]. The use of the internet is the first step in collecting information, with the main component being the internet service provider (ISP) [2]. ISP is a business entity or company engaged in internet service [3]. Without an ISP, the process of sending and receiving data via the internet network cannot be carried out efficiently. The reverse charging model is a model for recognizing the quality of service and speed of user access [4]. The reverse charging scheme focuses on charging that is only done in one direction, namely by one ISP to the ISP's customers, so that it is not possible for other ISPs to do reverse charging [5]. ISPs need to provide optimal quality of service (QoS) and offer profitable internet financing schemes, with three pricing models (flat fee, usage based, and two-part tariff) [6].

Demand response (DR) is an adjustment of internet usage based on consumer behavior that adapts to changes in internet prices or incentives [7]. DR programs are divided into price-based and incentive-based programs. While utility can be defined as a preference for formulating a decision regarding a certain value by minimizing risk [8]. Consumers adjust internet usage due to price changes or incentives [9]. DR programs are divided into price-based and incentive-based programs. Utility is defined as a preference for making decisions based on certain values. Decision making in utility aims to minimize risk [10]. Price-based programs are a DR strategy that uses tariff changes to influence consumer behavior [11]. Incentive-based programs are a DR approach that rewards consumers who change their internet usage patterns [12].

Incentives in DR can be in the form of discounts, quota bonuses, or additional service [13]. There are several types of price-based DR programs that are commonly applied in the internet service industry. Time-of-use pricing is a DR program that sets different rates during peak and low hours [14]. Critical peak pricing is a DR strategy where rates increase sharply when there is a spike in usage [15]. Real-time pricing is a DR scheme with prices that change every hour according to network conditions [16], [17]. Price-based demand response is an important strategy to facilitate ISPs and users' access to information and services while providing users with incentives. An internet incentive financing model that uses a quasi-linear utility function with the assumption of a solution strategy. To achieve optimal results, an incentive scheme is needed that takes into account the diversity of consumer characteristics. This concept is known as heterogeneous incentives, which are designed to suit differences in consumer preferences, consumption patterns, and responses to the incentives offered. Heterogeneous incentives are aimed at heterogeneous consumers which are focused on developing Heterogeneous incentives for heterogeneous consumers so that ISPs can achieve maximum profits. This study was conducted for high-end consumers, a group of users who require high quality of service and are willing to pay more [18]. In the context of internet service pricing strategies, high-end consumer preferences aim to develop sustainable business models. Low-end consumers are a group of users looking for affordable internet services [19]. QoS is a measure of network performance that includes connection speed and stability [20]. Traffic measurement is the process of monitoring the amount of data flowing in an internet network [21]. Bundling is a strategy for combining several internet services in one package at a certain price, to optimize the information service financing scheme model by considering bundling, as well as combining quasi-linear utility functions and reverse charging models [22]. The modified model is applied to high-end and low-end heterogeneous consumers [23], [24]. The high-end consumer type is a group of users who require high service quality and are willing to pay more, and the low-end consumer type is a group of users who seek affordable internet services.

The modified model is a development of the basic model with parameter and variable adjustments to reflect more realistic market conditions [25]. This development model is modeled in the form of mixed integer nonlinear programming (MINLP). MINLP is a mathematical optimization technique used to solve complex problems with discrete (integer) and continuous variables [26]. This development model is solved with the LINGO 13.0 application. The purpose of this study is to develop and determine the optimal solution of the improved incentive financing model for DR-based information services and heterogeneous incentives using a combination of reverse charging models and quasi-linear utility functions, and considering bundling in three information service financing schemes for heterogeneous consumers. The financing scheme for information services is a flat fee, usage-based, and two-part tariff.

2. METHOD

The proposed information service financing model utilizes a bundling scheme combined with a demand response-based reverse charging model and heterogeneous incentives on wired and wireless services according to a multi-service network. This service financing model is also modified based on a quasi-linear utility function for three financing schemes, namely: flat-fee, usage-based, and two-part tariff. Defining parameters and decision variables is necessary in developing an incentive financing model to measure the level of bandwidth consumption on the network used. Development of the financing model was completed with LINGO 13.0 software. The representation of the research method of the information service financing development model can be seen in Figure 1.

3. RESULTS AND DISCUSSION

Digilib traffic data is obtained from one of the local servers consisting of two components, namely sending data (inbound) and receiving data (outbound), expressed in bytes per second. Data retrieval was carried out over a period of 1 month (30 days), namely from March 2024 to April 2024. Table 1 and Table 2 present data on multi-class QoS network usage obtained from one of the local servers, namely, digilib traffic data. The data is divided into two categories, namely data during peak hours (starting at 07.00 and ending at 17.00 West Indonesia Time=WIT) and data during off-peak hours (starting at 17.00 and ending at 06.00 WIT). The network data collection is summed up, and then the average is taken every second for both inbound and outbound data for daily use use as in Tables 1 and 2. Traffic data usage during peak and off-peak hours is shown in Table 3. The notations for several parameters and variables used in the development of this incentive model are shown in Tables 4 and 5. Detailed parameter value determination is shown in Table 6.

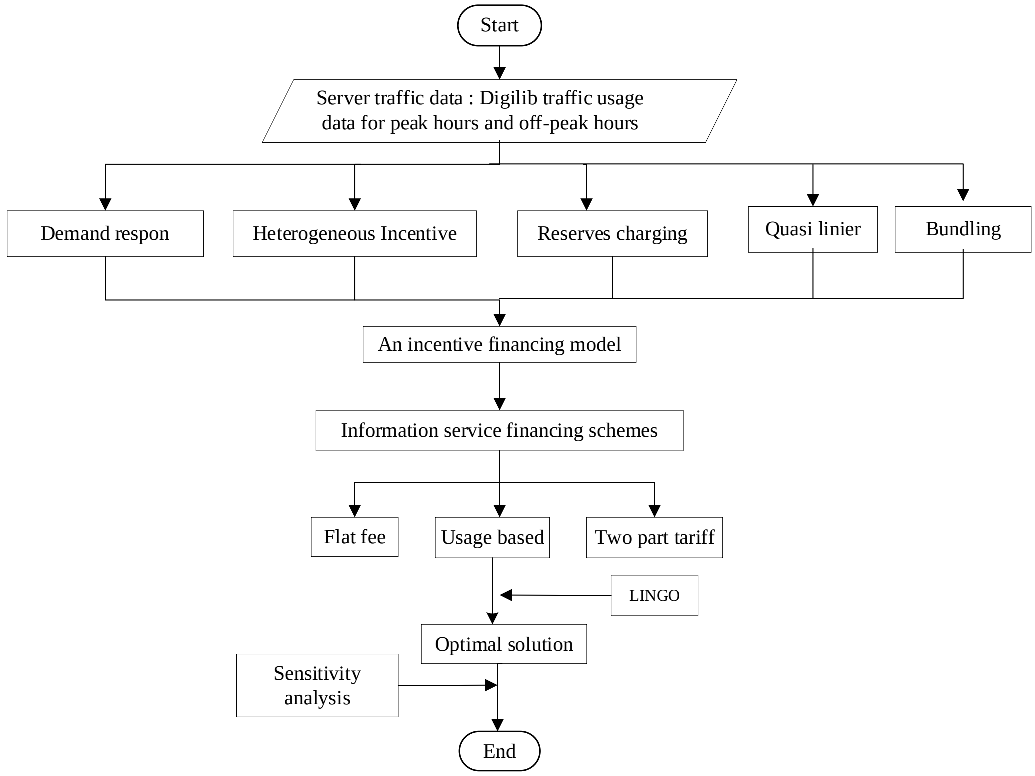


Figure 1. Flowchart of information service financing development model

Table 1. Traffic data for multi class QoS networks at peak hours

No	Date	Traffic (in bits per second) received	send	Total usage per day
1	16 March 2024	588,390	3,216,555	3,804,945
2	17 March 2024	428,776	3,721,798	4,150,574
3	18 March 2024	323,232	3,558,816	3,882,048
4	19 March 2024	314,986	2,773,013	3,087,999
5	20 March 2024	445,091	8,482,020	8,927,112
6	21 March 2024	694,288	6,727,641	7,421,930
7	22 March 2024	478,192	2,634,479	3,112,671
8	23 March 2024	502,340	2,182,787	2,685,127
9	24 March 2024	356,391	2,578,599	2,934,990
10	25 March 2024	315,280	2,929,009	3,244,290
11	26 March 2024	236,593	3,026,798	3,263,391
12	27 March 2024	830,283	5,348,120	6,178,403
13	28 March 2024	224,401	3,960,449	4,184,850
14	29 March 2024	323,997	2,692,686	3,016,684
15	30 March 2024	425,655	2,118,759	2,544,414
16	31 March 2024	219,100	2,434,244	2,653,344
17	01 April 2024	317,283	4,062,895	4,380,178
18	02 April 2024	198,250	3,704,336	3,902,586
19	03 April 2024	706,009	4,060,566	4,766,575
20	04 April 2024	351,797	3,942,987	4,294,784
21	05 April 2024	268,398	2,445,886	2,714,284
22	06 April 2024	231,469	3,277,091	3,508,560
23	07 April 2024	803,014	3,710,156	4,513,170
24	08 April 2024	443,089	4,717,149	5,160,238
25	09 April 2024	464,351	7,608,905	8,073,256
26	10 April 2024	208,911	2,647,284	2,856,195
27	11 April 2024	506,522	2,719,462	3,225,984
28	12 April 2024	284,182	1,000,008	1,284,190
29	13 April 2024	243,013	1,726,439	1,969,452
30	14 April 2024	260,976	1,924,345	2,185,322
31	15 April 2024	236,416	2,180,459	2,416,875
	Demand	12,230,673	108,113,745	120,344,418

Table 2. Traffic data for multi class QoS networks at off-peak hours

No	Date	Traffic (in bits per second)		Total usage per day
		Received	Send	
1	16 March 2024	129,163	1,151,348	1,280,511
2	17 March 2024	791,352	4,740,432	5,531,784
3	18 March 2024	438,082	272,0626	3,158,708
4	19 March 2024	1,262,476	11,401,717	12,664,192
5	20 March 2024	716,964	4,807,953	5,524,917
6	21 March 2024	751,301	3,079,185	3,830,486
7	22 March 2024	281,002	3,637,979	3,918,981
8	23 March 2024	669,433	7,762,574	8,432,007
9	24 March 2024	353,858	2,337,620	2,691,478
10	25 March 2024	336,837	3,494,788	3,831,625
11	26 March 2024	567,893	3,202,586	3,770,479
12	27 March 2024	312,807	3,040,768	3,353,575
13	28 March 2024	458,343	4,271,278	4,729,621
14	29 March 2024	335,423	4,483,154	4,818,577
15	30 March 2024	675,735	4,739,268	5,415,003
16	31 March 2024	387,666	5,479,669	5,867,335
17	01 April 2024	875,517	2,911,547	3,787,064
18	02 April 2024	763,964	4,996,546	5,760,510
19	03 April 2024	630,325	6,442,424	7,072,749
20	04 April 2024	580,439	5,534,384	6,114,823
21	05 April 2024	516,181	4,067,551	4,583,732
22	06 April 2024	251,435	2,990,710	3,242,145
23	07 April 2024	469,652	5,100,155	5,569,807
24	08 April 2024	359,748	4,195,608	4,555,356
25	09 April 2024	358,570	5,064,066	5,422,637
26	10 April 2024	557,998	3,133,900	3,691,899
27	11 April 2024	255,087	4,079,193	4,334,280
28	12 April 2024	195,011	2,038,432	2,233,443
29	13 April 2024	268,810	2,275,920	2,544,729
30	14 April 2024	418,469	3,131,572	3,550,042
31	15 April 2024	286,303	2,866,145	3,152,448
Demand		15,255,844	129,179,098	144,434,942

Table 3. Digilib traffic usage data for peak hours and off-peak hours

Consumption rate	Digilib usage (bytes)	Digilib usage (kilobytes)
$\bar{X} = \bar{X}_1$	8,927,112	8,717
$\bar{X}_2$	8,073,256	7,884
$\bar{Y} = \bar{Y}_1$	12,664,192	12,367
$\bar{Y}_2$	8,432,007	8,234
$\bar{X} = \bar{X}_1$	8,927,112	8,717

Table 4. Notation and description of parameters

Parameters	Description
α	Base price for each service
β	Premium quality for every service
C	Total capacity available on the network
KA_{kj}	Cost changes along with changes in the QoS available on service k and network j
b_{kj}	Capacity required in k services and networks j
p_{kj}	Consumer price of service k on network j
a_{kj}	Prices for consumers of service k on network j
$\bar{X}_w$	The maximum consumption rate of a consumer w on a service during peak hours
$\bar{Y}_w$	The maximum consumption level of consumer w on the service during off-peak hours.
G	Initial demand level function
$\hat{D}$	Initial demand level
$\tilde{D}$	Next level of demand
R_{wk}	Total order price for each consumer to w on each service to k
B_k	Internet demand rate function after considering the service model k
M	Marginal cost if adding more than one service

3.1. Development model of information service financing scheme during rush hour

This development model combines reverse billing and bundling models based on a quasi-linear utility function. Furthermore, it integrates demand response with heterogeneous incentives to account for variations in user preferences. This combination results in a more adaptive approach to information service financing that prioritizes customer satisfaction.

Table 5. Notation and description of variables

Variables	Description
$\pi\hat{B}$	Internet prices upon request
cX	The coefficient or utility unit value that shows how much benefit consumers get from each consumption unit X during peak hours
$h(Y)$	Additional utility that depends on the consumption of Y during off-peak hours
X	Consumer consumption levels on services during peak hours
Y	The maximum consumption level that can be achieved by consumers on off-peak service hours
Z	Decision variables that indicate whether to choose to participate in the service or not
P	Costs incurred by consumers to use the service
P_X	Unit charges applied by ISPs during peak hours
KA_{kj}	Unit charges applied by ISPs during off-peak hours
KB_{kj}	The cost of connecting with the QoS available on service k and network j
T_w	Usage income for consumers to w
G_k	The price set for each bundle of services k
M	Marginal cost of adding more than one service
X_w	Consumption level w on services during peak hours
P_Y	Additional fees that must be paid if consumers choose to join the service program
PZ	Cost changes along with changes in the QoS available
Y_w	Consumer consumption level w on services during off-peak hours
X_{kj}	Consumer price of service k on network j
π^a	Consumer consumption level w on services during peak hours
$\dot{E}$	Price elasticity of demand
e	Fixed unit costs
T_l	Traffic load
L_x	Linearity factor
H_p	Unit price received
Q	Quantity produced
Q_i	Quantity produced in period k
q_i	Amount produced at start k
l_k	The minimum base price required for the service k
B	Set linear parameters

Table 6. Peak and off-peak traffic data parameter values

Parameter	Value	Parameter	Value	Parameter	Value
α	0.1	b_{kj}	117.52	$\bar{Y}_1$	12,367
β	0.5	b_{kj}	141.05	$\bar{X}_2$	7,884
C	350,000	a_{11}	0.15	$\bar{Y}_2$	8,234
KA_{11}	0.5	a_{12}	0.15	G	1,000
KA_{12}	0.5	a_{21}	0.14	$\hat{D}$	100
KA_{21}	0.6	a_{22}	0.14	$\tilde{D}$	500
KA_{22}	0.6	T_1	50	R_{11}	1,000
p_{11}	10	T_2	1,000	R_{12}	2,150
p_{12}	10	a_{11}	0.05	R_{21}	1,130
p_{21}	45	a_{12}	0.05	R_{22}	300
p_{22}	45	a_{21}	0.06	B_1	200
p_{21}	15	a_{22}	0.06	B_2	15
p_{22}	15	$\bar{X}_1$	8,717	M	200

3.1.1. Case 1: KB_{kj} increase x increase

The information service financing model with peak hour traffic data for case 1 is arranged in three financing schemes as follows. The aim of this research is to maximize ISP profits with an incentive financing model in an information service financing scheme that considers quasi-linear functions for the case of KB_{kj} increases and x decreases, as defined by (1). Constraint (2) states that the total quantity produced must be less than the initial production. Constraint (3) states that the quantity produced must equal the total cost received. Constraint (4) states that the available capacity must be greater than the required network cost. Constraint (5) states that the quantity in production period k must be greater than the quantity produced at the beginning of k. Constraints (6) and (7) are 0 for their consumption levels (X_w and Y_w). If consumer w chooses to join the program and chooses Z_w=1, then the consumer must determine the optimal consumption levels of X_w and Y_w, which do not exceed the upper bounds of $\bar{X}_w$ and $\bar{Y}_w$. Constraint (8) states that ISP considers quasi-linear functions that have non-negative values. Constraint (9) is determined by consumer w, in this case it is assumed that consumer 1 chooses to join the program or Z_1 is 1. In an effort to maximize the level of consumer satisfaction, a flat fee financing scheme was established, such as (10) to (12), a two-part tariff financing scheme such as (13) to (15), and a usage-based financing scheme such as (16) to (18).

$$\text{Max } R = \sum_{i=1}^{2} H_p Q - \dot{e}(Q_i - q_i)^2 + G - \pi\hat{B} - cX + Y^b - P_X X - P_Y Y - PZ + \sum_{k=1}^{2}\sum_{j=1}^{2}\left(\left(KA_{kj} + KB_{kj}\right) + (\alpha + \beta l_k)p_{kj}x_{kj}\right) + \sum_{w=1}^{2}\sum_{k=1}^{2}(G_k - B_k)W_{wk} - \sum_{k=1}^{2} MY_k \quad (1)$$

Subjects to:

$$D_i \geq Q - q_i \quad (2)$$

$$Q_i - q_i = \frac{H_p}{2\dot{e}} \quad (3)$$

$$\frac{H_p}{Q_i - q_i} < 2\dot{e} \quad (4)$$

$$Q_i \geq q_i \quad (5)$$

$$X_w \leq \bar{X}_w Z_w \text{ ; } w = 1, 2 \quad (6)$$

$$Y_w \leq \bar{Y}_w Z_w; \; w = 1, 2 \quad (7)$$

$$2X + Y^4 - P_x X - P_Y Y - PZ \geq 0 \quad (8)$$

$$Z_1 = 1 \quad (9)$$

for the flat fee scheme

$$P_X = 0 \quad (10)$$

$$P_Y = 0 \quad (11)$$

$$P > 0 \quad (12)$$

for the two-part tariff scheme

$$P_X > 0 \quad (13)$$

$$P_Y > 0 \quad (14)$$

$$P = 0 \quad (15)$$

for the usage-based scheme

$$P_X > 0 \quad (16)$$

$$P_Y > 0 \quad (17)$$

$$P > 0 \quad (18)$$

Constraint (19) indicates that the required service capacity cannot exceed the reserved network capacity. Constraint (20) states that the required service capacity cannot exceed the total network capacity in network j. Constraint (21) states that the network capacity has a different allocation for each service, with a value of 0 or 1. Constraint (22) states that the QoS level is within a predetermined range for each service, with the QoS level being a positive integer. Constraint (23) states that consumers implement non-negative service usage and cannot exceed the service usage amount set by the ISP. Constraint (24) explains that cost changes depend on cost factors involving the QoS attribute Bandwidth, with base costs in service k and network j, and a linearity factor. Inequalities, the sign difference is caused by an increase or decrease in the QoS value. Constraint (25) describes the base cost for a connection to service k and network j, $(e - e^{-xB})$ and the traffic load. Constraint (26) describes the linearity factor that depends on parameters α and $(e - e^{-xB})$. Constraint (27) describes the linearity factor a_{kj}'s value, which is within the ISP-set value during peak hours. Constraint (28) describes the linearity factor a_{kj}'s value, which is within the ISP-set value during off-peak hours. Constraint (29) explains the limit on the allowable traffic load, T_l which is also determined by the ISP.

Constraint (30) explains the number of QoS value increases or decreases, set to 0 and 1, implicitly indicating that 0 represents best effort and 1 represents perfect service. The value of the linear parameter B in Constraint (31) has been set to a range of 0.8 to 1.07 because it is within this range that the best quality of service occurs. The value of the linear parameter a in Constraint (32) is a linear parameter defined by parameter a, with parameter a defining the base price level. Constraint (33) states that the function of the initial demand level for each consumer. Constraint (34) states that each consumer must maximize any surplus derived from the supplier's offered price for each bundle. Constraint (35) states that consumer surplus is defined as the difference between the price at which consumer w orders and the fixed price for each bundle issue. Constraint (36) states that each consumer will choose a bundle if they perceive the benefits of the bundled products. Constraint (37) states that consumers must acquire at least one bundled product and cannot make any purchases. Constraint (38) states that consumers can choose or not purchase the bundled product if the service provider offers bundled items. Constraint (39) states that consumer surplus is non-negative. Constraint (40) states that the value must be non-negative for the pricing set by each service package k. Constraint (39) states that the consumer surplus is non-negative. Constraint (40) states that the value must be non-negative for the pricing set by each service package k.

$$l_k b_{kj} x_{kj} \leq a_k C;\ k = 1, 2;\ j = 1, 2 \tag{19}$$

$$\sum_{j=1}^{2} l_k b_{kj} x_{kj} \leq a_k C\ ;\ j = 1, \tag{20}$$

$$\sum_{k=1}^{2} a_k = 1, \quad a_k \in \{0, 1\} \tag{21}$$

$$0.001 \leq l_k \leq 1;\ k = 1, 2 \tag{22}$$

$$0 \leq X_{kj} \leq 10;\ k = 1, 2;\ j = 1, 2 \tag{23}$$

$$KB_{kj} = \left(1 + \frac{x}{2000}\right) KC_{11} Lx;\ k = 1, 2;\ j = 1, 2 \tag{24}$$

$$KC_{kj} = a_{kj}(e - e^{-xB}) T_l / 100;\ k = 1, 2;\ j = 1, 2 \tag{25}$$

$$Lx = a(e - e^{-xB}) \tag{26}$$

$$0.05 \leq a_{1j} \leq 0.15;\ j = 1, 2 \tag{27}$$

$$0.06 \leq a_{2j} \leq 0.14;\ j = 1, 2 \tag{28}$$

$$50 \leq T_l \leq 1000 \tag{29}$$

$$0 \leq x \leq 1 \tag{30}$$

$$0.8 \leq B \leq 1.07 \tag{31}$$

$$a = 1 \tag{32}$$

$$G = (G^a + \pi^a)(\widehat{D} - \widetilde{D}) + \left(\frac{\pi^a}{2\dot{E}\widehat{D}}\right)(\widehat{D} - \widetilde{D})^2 \tag{33}$$

$$T_w \geq (R_{wk} - G_k) Y_k;\ w = 1, 2;\ k = 1, 2 \tag{34}$$

$$T_w = \sum_{w=1}^{w} (R_{wk} - G_k)\, W_{wk}\ ;\ k = 1, 2 \tag{35}$$

$$(R_{wk} - G_k) W_{wk} \geq 0;\ w = 1, 2;\ k = 1, 2 \tag{36}$$

$$\sum_{w=1}^{2} W_{wk} \leq 1;\ k = 1, 2 \tag{37}$$

$$W_{wk} \leq Y_k;\ w = 1, 2;\ k = 1, 2 \tag{38}$$

$$T_w \geq 0;\ w = 1, 2 \tag{39}$$

$$G_w \geq 0;\ w = 1, 2 \tag{40}$$

3.1.2. Case 2: KB_{kj} increase x decrease

Objective function (1) and constraints (2) to (22) continued with additional constraints. Continued with constraint (24) to (40). Constraint (41) states that the cost of establishing a connection with the available QoS on service k and network j for the case KB_{kj} increases and x decreases.

$$KB_{kj} = \left(1 - \frac{x}{2000}\right) KC_{kj} Lx;\ k = 1, 2;\ j = 1, 2 \quad (41)$$

3.1.3. Case 3: KB_{kj} decrease x increase

The information service financing model with peak hour traffic data for case 3 is arranged in three financing schemes as follows. Continued with constraint (24) to (40). The aim of this study is also to maximize ISP profits with an incentive financing model in an information service financing scheme that considers quasi-linear functions for the case of decreasing KB_{kj} and decreasing x, as defined by (42).

$$\text{Max } R = \sum_{i=1}^{2} H_p Q - \dot{e}(Q_i - q_i)^2 + G - \pi\hat{B} - cX + Y^b - P_X X - P_Y Y - PZ + \sum_{k=1}^{2}\sum_{j=1}^{2}\big((KA_{kj} - KB_{kj}) + (\alpha + \beta l_k) p_{kj} x_{kj}\big) + \sum_{w=1}^{2}\sum_{k=1}^{2}(G_k - B_k) W_{wk} - \sum_{k=1}^{2} MY_k \quad (42)$$

3.1.4. Case 4: KB_{kj} decrease x increase

Objective function (42) and constraints (2) to (22) continued with constraints (40) and continued with constraints (25) to (40).

3.2. Development model of information service financing scheme during off-peak hours

3.2.1. Case 1: KB_{kj} increase x increase

Objective function (1) and constraint (2) continued with constraints. Continued with constraint (8) to constraint (40). Constraint (43) ensures that the ISP optimizes consumer consumption of the service during peak hours. Constraint (44) ensures that the ISP maximizes consumer consumption of the service during off-peak hours.

$$X_2 \leq \bar{X}_2 Z_2 \quad (43)$$

$$Y_2 \leq \bar{Y}_2 Z_2 \quad (44)$$

3.2.2. Case 2: KB_{kj} decrease x increase

Objective function (1) and constraints (2) to (5) continued by constraints (43) to (44) and continued with constraints (8) to (23) and continued by constraint. Continued with Constraint (25) to (40). Constraint (45) states that the cost of establishing a connection with the available QoS on service k and network j for the case KB_{kj} decreases and x increases.

$$KB_{kj} = \left(1 - \frac{x}{2000}\right) KC_{11} Lx;\ k = 1, 2;\ j = 1, 2 \quad (45)$$

3.2.3. Case 3: KB_{kj} decrease x increase

The information service financing model with off peak hour traffic data for case 3 is arranged in three financing schemes as follows. Continued with constraint (2) to (40). The aim of this study is also to maximize ISP profits with an incentive financing model in an information service financing scheme that considers quasi-linear functions for the case of decreasing KB_{kj} and increasing x, as defined by (46).

$$\text{Max } R = \sum_{i=1}^{2} H_p Q - \dot{e}(Q_i - q_i)^2 + G - \pi\hat{B} - cX + Y^b - P_X X - P_Y Y - PZ + \sum_{k=1}^{2}\sum_{j=1}^{2}\big((KA_{kj} - KB_{kj}) + (\alpha + \beta l_k) p_{kj} x_{kj}\big) + \sum_{w=1}^{2}\sum_{k=1}^{2}(G_k - B_k) W_{wk} - \sum_{k=1}^{2} MY_k \quad (46)$$

3.2.4. Case 4: KB_{kj} decrease x decrease

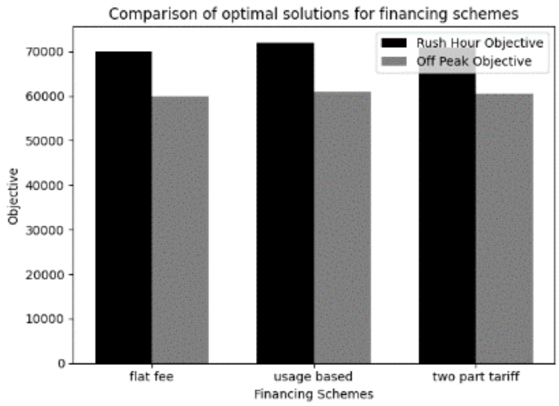
Objective function (46) and constraint (2) to (23) continued by constraints (24), then continued with constraints (25) to (40). The optimal solution of the information service financing scheme development model for each financing scheme with peak hour and non-peak hour traffic data for each financing scheme is shown in Tables 7 and 8. Comparison of optimal solutions in three financing schemes with traffic data during peak hours and data during non-peak hours, as shown in Figure 2.

Table 7. Comparison of optimal solutions on three financing schemes with traffic data rush hour

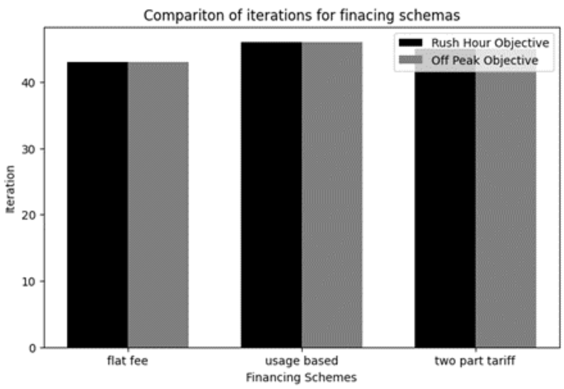
Solver status	Financing scheme		
	Flat fee	Usage based	Two part tariff
Model class	MINLP	MINLP	MINLP
State	Local optimal	Local optimal	Local optimal
Objective	70,715.69	70,715.69	70,715.69
Infeasibility	0	0	0
Iterations	43	46	46
Extended solver status			
Solver type	Branch and bound	Branch and bound	Branch and bound
Best objective	70,715.69	70,715.69	70,715.69
Steps	0	0	0
Update interval	2	2	2
GMU (K)	46	46	46
ER (Sec)	0	0	0

Table 8. Comparison of optimal solutions on three financing schemes with traffic data off-peak hours

Solver status	Financing scheme		
	Flat fee	Usage based	Two part Tariff
Model class	MINLP	MINLP	MINLP
State	Local optimal	Local optimal	Local optimal
Objective	59,545.78	59,545.78	59,545.78
Infeasibility	0	0	0
Iterations	43	46	46
Extended solver status			
Solver type	Branch and Bound	Branch and Bound	Branch and Bound
Best objective	59,545.78	59,545.78	59,545.78
Steps	0	0	0
Update interval	2	2	2
GMU (K)	54	46	46
ER (Sec)	0	0	0



(a)



(b)

Figure 2. Results of three financing schemes with (a) traffic data during peak hours and (b) data during non-peak hours

3.3. Sensitivity analysis and equilibrium analysis

Sensitivity analysis is a systematic method to examine the extent to which changes in variables can affect the results or optimal solutions of a mathematical model, with the aim of assessing the reliability of the solution [27]. In the flat fee scheme, the variable \pi has a nonlinear current coefficient, the allowable increase is 0, and the allowable decrease is ∞. In the two-part tariff scheme, the variables P_X and P_Y have a nonlinear current coefficient, the allowable increase is ∞, and the allowable decrease is 0. In the usage-based scheme, the variable P_Y has a nonlinear current coefficient, the allowable increase is ∞, and the allowable decrease is 0.

Equilibrium analysis of the incentive financing development model for demand response-based information services and bundling in the information service financing scheme produces the following strategies. High-end consumers and low-end consumers who are looking for affordable internet services by applying an internet incentive financing model based on demand response and bundling that considers quasi-linear functions during peak hours obtained an optimal solution of IDR 70,715.69/kbps with 43 iterations, and based on consumer traffic data outside peak hours, an optimal solution was obtained of IDR 59,545.78/kbps with 43 iterations. Providers can apply an incentive financing development model for information services based on demand response and bundling in information service financing schemes by considering quasi-linear functions by combining several services in one package at a more competitive price, as well as improving information services.

The reverse charging model uses an additional bundling model based on demand response and heterogeneous incentives on a quasi-linear utility function compared to the model improved incentive pricing-based quasi-linear utility function [24], which is an improved model of incentive pricing based on internet usage in the reverse charging scheme based on a quasi-linear utility function. Model [24] has an optimal value of 652,427/kbps with 95 iterations, and our proposed model has been improved to 70,715.69/kbps with 43 iterations. The development of a reverse charging model using additional demand response-based bundling models and heterogeneous incentives on a quasi-linear utility function is better for achieving maximum ISP revenue that produces quality service and consumer access speed, so that the financing model with a quasi-linear utility function can be used as a recommendation for ISPs to optimize revenue and improve the quality of service to consumers.

4. CONCLUSION

The internet incentive financing model produces an optimal solution obtained by consumers of traffic data during peak hours and off-peak hours, namely by using a flat fee financing scheme in case 1 for KB_{kj} increases x increases. The optimal solution during peak hours based on the internet incentive financing model for traffic data was obtained at IDR 70,715.69/kbps with 43 iterations. Meanwhile, the optimal solution for off-peak hours based on the internet incentive financing model for traffic data is obtained at IDR 59,545.78/kbps with 43 iterations. The ISP's advantage with an incentive financing model in an information service financing scheme that considers quasi-linear functions for the case of an increase of KB_{kj} and a decrease of x in peak hour traffic data is more optimal than that of off-peak hour traffic data. Further research is suggested to use other utility functions, such as stone-geary which has advantages in modeling minimum consumption and accommodating consumer preference heterogeneity better. This needs to be developed in order to produce an optimal bundling strategy for various consumer segments with different basic needs in information services. An internet incentive financing model that uses quasi-linear utility functions and additional bundling for heterogeneous consumers can be implemented in a smart grid system by relying on a dynamic pricing mechanism under uncertainty to formulate value-based decisions.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

REFERENCES

[1] J. N. Nduati, R. Mang'ana, "The influence of strategic leadership and competitiveness of internet service providers in Kenya," *International Academic Journal of Human Resource and Business Administration*, vol. 4, no. 4, pp. 53–72, 2024.

[2] H. Song, S. Han, W. Liu, and A. Ganguly, "What role do FinTech companies play in supply chain finance? a signaling intermediary perspective," *Journal of Business & Industrial Marketing*, vol. 38, no. 6, pp. 1279–1294, 2023, doi: 10.1108/JBIM-12-2021-0587.

[3] I. A. Omar, R. Jayaraman, K. Salah, M. Debe, and M. Omar, "Enhancing vendor managed inventory supply chain operations using blockchain smart contracts," *IEEE Access*, vol. 8, pp. 182704–182719, 2020, doi: 10.1109/ACCESS.2020.3028031.

[4] S. Cheng and H. Norcross, "Internet censorship in the time of a global pandemic: a proposal for revisions to section 230 of the communications decency act," *Brigham Young University Prelaw Review*, vol. 35, 2021.

[5] F. M. Puspita, A. Wulandari, E. Yuliza, R. Sitepu, and Yunita, "Modification of wireless reverse charging scheme with bundling optimization issues," in *2020 3rd International Seminar on Research of Information Technology and Intelligent Systems (ISRITI)*, pp. 556–561, 2020, doi: 10.1109/ISRITI51436.2020.9315348.

[6] K. Ngov, Y. Ko, J. Noh, S. Lee, and S. Han, "Fleet management for earthmoving operation in the phase of detail estimation using mixed integer nonlinear programming," *Journal of Asian Architecture and Building Engineering*, vol. 24, no. 2, pp. 800–815, 2024, doi: 10.1080/13467581.2024.2320321.

[7] E. Guelpa and V. Verda, "Demand response and other demand side management techniques for district heating: a review," *Energy*, vol. 219, 2021, doi: 10.1016/j.energy.2020.119440.

[8] F. M. Puspita, E. Yuliza, W. Herlina, Yunita, and Rohania, "Improved multi-service-reverse charging models for the multi-link internet wireless using bit error rate QoS attribute," *Science and Technology Indonesia*, vol. 5, no. 1, pp. 6–13, 2020, doi: 10.26554/sti.2020.5.1.6-13.

[9] J. Lee, S. H. Ha, H. Yoon, J. Baek, J. S. Kim, and G. W. Moon, "Reverse-pulse charging method of lithium batteries for stable charging at low temperatures," *Journal of Electrochemical Science and Technology*, vol. 16, no. 1, pp. 113–128, 2025, doi: 10.33961/jecst.2024.00969.

[10] M. Naji, S. Thiruchelvam, and M. Khudari, "The mediating role of the service provider selection on the relationship between internet service criteria and institution performance," *International Journal of Religion*, vol. 5, no. 10, pp. 3104–3119, 2024, doi: 10.61707/ha2vgr63.

[11] R. Mao, X. Li, "Optimizing product pool selection and pricing for size-based bundling promotions," *SSRN*, 2024.

[12] Z. Zhu *et al.*, "Surface plasmon mediates the visible light--responsive lithium--oxygen battery with Au nanoparticles on defective carbon nitride," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 118, no. 17, pp. 2–7, 2021, doi: 10.1073/pnas.2024619118.

[13] A. Bhardwaj, K. John, and S. Mukherjee, "Incentives of fund managers and precautionary fire sales," *SSRN Electronic Journal*, 2021, doi: 10.2139/ssrn.3952358.

[14] R. Hledik, A. Ramakrishnan, S. Patel, and A. Satchwell, "Distributed energy, utility scale: 30 proven strategies to increase VPP enrollment," *Berkeley Lab*, 2024. [Online]. Available: https://eta-publications.lbl.gov/publications/distributed-energy-utility-scale-30.

[15] T. Abdallah, A. Braverman, and W. Gu, "Multi-purchase assortment optimization under a general random utility model," *SSRN Electronic Journal*, 2024, doi: 10.2139/ssrn.4842012.

[16] C. Challoumis, "Comparative analysis between cost and liability based on the sensitivity method," *Open Journal of Sociological Studies*, vol. 8, no. 1, pp. 25–38, 2024, doi: 10.32591/coas.ojss.0801.03025c.

[17] T. Alquthami, A. H. Milyani, M. Awais, and M. B. Rasheed, "An incentive based dynamic pricing in smart grid: a customer's perspective," *Sustainability*, vol. 13, no. 11, pp. 1–17, 2021, doi: 10.3390/su13116066.

[18] K. Bestuzheva, A. Chmiela, B. Müller, F. Serrano, S. Vigerske, and F. Wegscheider, "Global optimization of mixed-integer nonlinear programs with SCIP 8," *Journal of Global Optimization*, vol. 91, no. 2, pp. 287–310, 202AD.

[19] A. Lundell and J. Kronqvist, "Polyhedral approximation strategies for nonconvex mixed-integer nonlinear programming in SHOT," *Journal of Global Optimization*, vol. 82, no. 4, pp. 863–896, 2022, doi: 10.1007/s10898-021-01006-1.

[20] A. Chandra, A. Moinat, and H. Weber, "A priori bounds for the Φ4 equation in the full sub-critical regime," *Archive for Rational Mechanics and Analysis*, vol. 247, no. 3, pp. 71–118, 2023, doi: 10.1007/s00205-023-01876-7.

[21] Y. Alnafisah, F. Masood, A. Muhib, O. Moaaz, "Improved oscillation theorems for even-order quasi-linear," *Symmetry*, vol. 15, no. 5, pp. 1–25, 2023, doi: 10.3390/sym15051128.
[22] P. Linares and F. Otto, "A tree-free approach to regularity structures: the regular case for quasi-linear equations," *arXiv:2207.10627*, 2022.
[23] F. Fischer, "A non-local quasi-linear ground state representation and criticality theory," *Calculus of Variations and Partial Differential Equations*, vol. 62, no. 5, pp. 1–33, 2023, doi: 10.1007/s00526-023-02496-5.
[24] F. M. Puspita, B. J. Rezky, A. N. Y. Simarmata, E. Yuliza, and Y. Hartono, "Improved incentive pricing-based quasi-linear utility function of wireless networks," *Indonesian Journal of Electrical Engineering and Computer Science*, vol. 22, no. 3, pp. 1467–1475, 2021, doi: 10.11591/ijeecs.v22.i3.pp1467-1475.
[25] Z. Dehvari, S. M. S. M. Mosadegh, "The stability of functional equations in quasi-normed quasilinear spaces," *Journal of Mathematical Extension*, vol. 17, no. 8, pp. 1–14, 2023, doi: 10.30495/JME.2023.2611.
[26] A. J. Brizard and A. A. Chan, "Hamiltonian formulations of quasilinear theory for magnetized plasmas," *Frontiers in Astronomy and Space Sciences*, vol. 9, pp. 1–14, 2022, doi: 10.3389/fspas.2022.1010133.
[27] U. Krishnan and M. Masrom, "Optimizing transportation cost using linear programming: a Malaysian case study," *Open International Journal of Informatics*, vol. 10, no. 1, pp. 1–12, 2022, doi: 10.11113/oiji2022.10n1.171.

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