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Soft fuzzy partial metric and some results on fixed point theory under soft set

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ABSTRACT

This research paper establishes a new concept of soft fuzzy partial metric spaces, combining soft sets, partial metric spaces, and fuzzy sets to handle uncertainty and imprecision. This paper's primary goal is to use soft fuzzy partial metric spaces to examine various fixed-point theory conclusions. A few fixed-point results are defined under the Ψ −contraction mapping on soft fuzzy partial metric space and the soft fuzzy contraction mapping. Also, illustrate the related example of fixed-point theorem. Soft fuzzy partial metric spaces have applications in various fields, including image processing, decision-making, and network analysis.

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1. INTRODUCTION

According to Zadeh's fuzzy set theory, elements can have degrees of membership in [0, 1] instead of only "in" or "out" [1]. He also generalized operations like union, intersection, and complement for fuzzy sets, and established properties of fuzzy relations and convex fuzzy sets. Kramosil and Michálek [2] introduced fuzzy metric spaces, which combine notions of distance fuzzily with an extra parameter and satisfy analogues of the triangle inequality. Presented fundamental results in fuzzy metric spaces, including convergence and Cauchy sequence characterizations [3].

Matthews introduced the partial metric spaces, which allow a point to have a non-zero distance to itself [4]. This relaxed form of metric is useful in computer science and has interesting topological consequences. In order to deal with uncertain, parameterized data, Molodtsov [5] suggested soft sets. This concept allows attribute-based parametrization of element membership without some of the drawbacks of fuzzy or rough sets. examines and contrasts the relationships between fuzzy soft sets, rough soft sets, and soft sets [6]. Useful for seeing how different uncertainty models overlap and differ. Investigates fuzzy soft metric spaces combining fuzzy set membership and soft set parameterization in a metric-like structure; studies basic structure like convergence, continuity, so that later fixed point or topological properties can be developed [7].

Introduces multi-fuzzy soft sets allowing multiple membership levels or fuzziness types under soft set parameters [8], and applies these to the decision making, demonstrating their utility in modelling uncertain preference or attribute-based judgments. Amer [9] defines fuzzy partial metric spaces allowing non-zero self-distance, but with fuzziness. Explores the structure, definitions, and often proving fixed point, completeness type results. Defines a soft fuzzy metric space, examines basic properties like convergence, mapping behavior, and possibly fixed-point existence [10].

Defines fuzzy soft metric and shows how one can induce a fuzzifying soft topology from it. Studies notions like open sets, convergence in this new topology [11]. Defines a few fixed-point theorems under different mapping or contraction conditions in partial fuzzy metric spaces [12]. Combines fuzzy and soft features with the G-metric to extend fixed point theory to fuzzy soft G-metric spaces [13]. Additional developments in fuzzy partial metric spaces include additional classes of mappings, improved conditions, and greater generality in fixed point theorems [14], [15]. Proposes neutrosophic soft metric spaces [16], studies soft compatible mappings and derives common fixed-point theorems in soft S-metric spaces [17], and explores convergence in partial soft metric spaces, establishing fundamental features required for additional fixed-point conclusions [18]. Contributed to fixed-point applications and soft topology by introducing results for soft B-metric spaces [19]. Improved computational thinking by applying soft set theory to decision-making problems through a soft AND-operation approach [20]. Studied fixed-point theory applications of the metrization of soft metric spaces [21]. Fixed-point theorems in soft parametric metric spaces were proved using C-class functions [22]. Expanding on fuzzy contraction concepts, fixed-point findings in soft B-fuzzy metric spaces were presented [23]. Fixed-point solutions with practical applications in modified intuitionistic fuzzy soft metric spaces were presented [24], proving existence and uniqueness results by examining ϕ-contraction mappings under soft fuzzy metric spaces [25]. Established common fixed-point theorems under the equiv-asymptotic (E. A.) property condition in fuzzy partial metric spaces [26].

The concept of soft fuzzy partial metric spaces is presented in this study along with an examination of their fundamental characteristics. We expand traditional conclusions to this generalized case by establishing new fixed-point theorems under different contractive conditions. Our contributions aim to deepen the theoretical understanding of fixed-point phenomena in soft fuzzy environments and to provide practical tools for applications where uncertainty, fuzziness, and parameter dependence coexist.

2. PROPOSED METHOD

In order to develop a novel concept of soft fuzzy partial metric space and related fixed-point theory, we describe certain fundamental definitions and properties of metric spaces and soft sets in this part.

2.1. Definition 2.1

A partial metric space on ‘$\mathbb{X}$’ is a pair (‘$\mathbb{X}$’, P) such that ‘$\mathbb{X}$’ is a non-empty set and P: ‘$\mathbb{X}$’ × ‘$\mathbb{X}$’ → $\mathbb{R}+$ is a mapping providing the listed conditions $\forall \mathcal{p}, \mathcal{q}, \mathcal{r} \in$ ‘$\mathbb{X}$’ such that:

i) ‘P‘$P(\mathcal{p},\mathcal{p}) \leq P(\mathcal{p},\mathcal{q})$’

ii) ‘$P(\mathcal{p},\mathcal{p}) = P(\mathcal{q},\mathcal{q}) = P(\mathcal{p},\mathcal{q})$’ if ‘$\mathcal{p} = \mathcal{q}$’

iii) ‘$P(\mathcal{p},\mathcal{q}) = P(\mathcal{q},\mathcal{p})$’

iv) ‘$P(\mathcal{p},\mathcal{r}) \leq P(\mathcal{p},\mathcal{q}) + P(\mathcal{q},\mathcal{r}) - P(\mathcal{q},\mathcal{q})'$

Note that a point's self-distance does not always equal 0 in partial metric space. The partial metric ‘P’ is an ordinary metric on ‘$\mathbb{X}'$ if ‘$P(\mathcal{p},\mathcal{p}) = 0'$, $\forall \mathcal{p} \in$ ‘$\mathbb{X}$’. So, a partial metric is an extension of an ordinary metric [4].

2.2. Definition 2.2

If the following criteria are satisfied, a binary operation "ʘ’" on [0, 1] is referred to as a continuous t-norm: $\forall \mathcal{p}, \mathcal{q}, \mathcal{r}, \mathcal{s} \in$ [0, 1]:

i) ‘$\mathcal{p} \odot \mathcal{q} = \mathcal{q} \odot \mathcal{p}$’ and ‘$\mathcal{p} \odot (\mathcal{q} \odot \mathcal{r}) = (\mathcal{p} \odot \mathcal{q}) \odot \mathcal{r}$’

ii) ‘ʘ is continuous on [0, 1]×[0, 1]’

iii) ‘$\mathcal{p} \odot 1 = \mathcal{p}$’

iv) If ‘$\mathcal{p} \leq \mathcal{q}$’ and ‘$\mathcal{r} \leq \mathcal{s}$’, then ‘$\mathcal{p} \odot \mathcal{r} \leq \mathcal{q} \odot \mathcal{s}$’ [2]

2.3. Definition 2.3

Considering ‘$\mathbb{X}$’ be a non-empty set, ‘ʘ’ be a continuous t-norm, and F: ‘$\mathbb{X}'$ × ‘$\mathbb{X}'$ × $(0,\infty) \rightarrow [0,1]$ be a mapping. Consider ‘F’ be a fuzzy set. If the specified conditions are satisfied $\forall \mathcal{p}, \mathcal{q}, \mathcal{r} \in$ ‘$\mathbb{X}$’ and $u, v > 0$, then the triplet (‘$\mathbb{X}$’, $F, \odot$) is said to be a fuzzy metric space, if it satisfies the subsequent properties for:

i) ‘$F(\mathcal{p},\mathcal{q}u) > 0$’,

ii) ‘$F(\mathcal{p},\mathcal{q}u) = 1$’, if ‘$\mathcal{q} = \mathcal{p}$

iii) ‘$F\ (\mathcal{p},\mathcal{q}u) = F(\mathcal{q},\mathcal{p},u)$’,

iv) ‘$F\ (\mathcal{p},\mathcal{q}u+v) \geq F(\mathcal{p},\mathcal{q}u) \odot F(\mathcal{p},\mathcal{q}v)$’

v) ‘$F\ (\mathcal{p},\mathcal{q}\odot)$, is continuous on $(0,\infty)$.’

If (‘$\mathbb{X}$’, $F, \odot$) is a fuzzy metric space, then ‘F’ is a fuzzy metric on ‘$\mathbb{X}$’ [2].

2.4. Definition 2.4

Considering ‘$\mathbb{X}$’ be a non-empty set, ‘ʘ’ be a continuous t-norm and F_p : ‘$\mathbb{X}' \times$ ‘$\mathbb{X}' \times [0,\infty) \to [0,1]$ be a mapping. Considering P be a partial metric space. If the specified conditions are satisfied $\forall \wp, q, r \in$ ‘$\mathbb{X}$’ and $u, v \geq 0$, then the triplet (‘$\mathbb{X}$’, $F_p, \odot$), is said to be a fuzzy partial metric space:

i) ‘$F_p(\wp, q0) = 0$’

ii) ‘$F_p(\wp, qu) = F_p\ (q, \wp, u)$’

iii) ‘$F_p(\wp, r, u+v) \geq F_p(\wp, qu) \odot F_p(q\ r, v)$’

iv) ‘$F_p(\wp, q, u) \leq 1, u > 0$’&‘$F_p(\wp, q\ , u) = 1$’ if ‘$p(\wp, q = 0$’

v) '$F_p\ (\wp, q, \odot)$,:(0, ∞)→[0, 1] is continuous

Where ‘$F_p(\wp, q, u) = \frac{u}{u+p(\wp,q)}$’, if (‘$\mathbb{X}$’, $F_p, \odot$), represents a fuzzy partial metric space, then ‘F_p’ denotes fuzzy partial metric on ‘$\mathbb{X}$’ [9]. Note: in this research paper, U refers universal set, $\wp$, which is the set of parameters, $P(U)$ is the power set of U. We define $\mathbb{K}\wp$ as the absolute soft set over U with parameter set $\wp$. $\mathfrak{B}(\mathfrak{R})$ is all non-void bounded subset of $\mathfrak{R}$ that is a collection of all real numbers.

2.5. Definition 2.5

A soft set $(F, \wp)$ over a universal set U is a pair, where $\wp$ is a set of parameters. F is a mapping given by $F: \wp \to P(U)$, where $P(U)$ represents power set of U. Put differently, for every parameter $e \in \wp$, F(e) is a subset of the universal set U [5].

2.5.1. Example 2.5.1

The example illustrates the definition of a soft set (F, $\wp$) over a universal set U. The theoretical framework involves a function F that maps each parameter in the set of parameters $\wp$, which is a subset of the universal set U. It means that power set of universal set.

Consider universal set: $U = \{a, b, c, d, e\}$ (set of houses)

Parameters: $\wp =$ {expensive, beautiful, modern} (set of parameters)

Define $(F, \wp)$ is a soft set over U, where:

$$F(expensive) = \{a, c\}\ , F(beautiful) = \{b, d\}, F(modern) = \{a, e\}$$

The soft set (F, $\wp$) can be interpreted as:

– $\{a, c\}$ are expensive houses
– $\{b, d\}$ are beautiful houses
– $\{a, e\}$ are modern houses.

This example shows how complex systems with many parameters can be modelled using soft sets.

2.6. Definition 2.6

In soft set theory, an absolute soft set is the “maximum’ possible soft set over a universal U, given a set of parameters $\wp$. It is essential for defining the basic logical operations that allow soft set theory to function as a mathematical tool. A soft set $(F, \wp)$ over a universal set U is referred to as an absolute soft set if $F(\mathfrak{n}) = \mathcal{K}, \forall\ \mathfrak{n} \in \wp$ [5].

2.6.1. Example 2.6.1

The example illustrates the theoretical concept of an absolute soft set, which is a specific type of soft set with particular properties. It shows that each parameter maps to a singleton set containing only itself, and the union of all resulting subsets covers the entire universal set.

Consider universal set: $U = \{a, b, c, d\}$

Define $(F, \wp)$ is an absolute soft set over U, where:

$$\wp = U\ = \{a, b, c, d\}$$

$$F(a) = \{a\}, F(b) = \{b\}, F(c) = \{c\}, F(d) = \{d\}$$

The absolute soft set $(F, \wp)$ satisfies:

i) $F(e) = \{e\}\ for\ all\ e \in U$

ii) $\cup\, F(e) = U$

Let's calculate the union of $F(a)$ and $F(b)$:

$$F(a) \cup F(b) = \{a\} \cup \{b\} = \{a, b\}$$

This example illustrates concept of absolute soft sets as well as their properties.

2.7. Definition 2.7

A null soft set $F(\mathfrak{n})$ is the foundational or empty object in soft set theory. It represents a scenario where none of the chosen parameters can be applied to any of the objects in the universe. Here, considering a function F, set of parameters $\wp$ which is subset of universal set $U, \mathfrak{n}$ is element set of parameters for defining a null soft set. A soft set $(F, \wp)$ over a universal set U is called a null soft set $F(\mathfrak{n}) = \{\ \}, \forall\ \mathfrak{n} \in \wp$ [10].

2.8. Definition 2.8

A pair $(F, \wp)$ is a soft real set if $F: \wp \rightarrow \mathfrak{B}(\mathfrak{R})$, where $\mathfrak{B}(\mathfrak{R})$ is all non-void bounded subsets of $\mathfrak{R}$ (collection of all real numbers). A soft real set (F, $\wp$) is a soft real number, if $\forall\ \mathfrak{n} \in \wp, F(\mathfrak{n})$ is a singleton member of $\mathfrak{B}(\mathfrak{R})$. For a soft real number $\mathfrak{x}$, if $\mathfrak{x}(\mathfrak{n}) = \{u\}, u > 0$ [10].

2.9. Definition 2.9

For two soft real numbers $\mathfrak{u}, \mathfrak{v}$, subsequent operations are as [10]:

i) $(\mathfrak{u} \oplus \mathfrak{v})(\mathfrak{n}) = \{\mathfrak{u}(\mathfrak{n}) + \mathfrak{v}(\mathfrak{n})/\mathfrak{n} \in \wp\}$

ii) $(\mathfrak{u} \ominus \mathfrak{v})(\mathfrak{n}) = \{\mathfrak{u}(\mathfrak{n}) - \mathfrak{v}(\mathfrak{n})/\mathfrak{n} \in \wp\}$

iii) $(\mathfrak{u} \otimes \mathfrak{v})(\mathfrak{n}) = \{\mathfrak{u}(\mathfrak{n}).\mathfrak{v}(\mathfrak{n})/\mathfrak{n} \in \wp\}$

2.10. Definition 2.10

The collection of ordered pairs,
$\mathcal{S}_{\mathfrak{f}} = \left\{(\mathscr{p}_i^{\rho 0}, \mu_{\mathcal{S}_{\mathfrak{f}}}(\mathscr{p}_i^{\rho 0})\ \right)/(\mathscr{p}_i^{\rho 0} \in \mathbb{K}\wp, \mathscr{p}_i^{\rho} \in \wp\}$ is a soft fuzzy set in $\mathbb{K}\wp$ as $\mu_{\mathcal{S}_{\mathfrak{f}}}$ is called a soft membership function define as $\mu_{\mathcal{S}_{\mathfrak{f}}}: \mathbb{K}\wp \rightarrow [0,1]\wp$. Thus, $\mu_{\mathcal{S}_{\mathfrak{f}}}(\mathscr{p}_i^{\rho 0})$ represents the associated soft membership grade of soft point $\mathscr{p}_i^{\rho 0}$ in $\mathcal{S}_{\mathfrak{f}}$ [10].

3. RESEARCH METHOD

3.1. Definition 3.1

Considering 'X' be a non-empty set, 'Θ' be a continuous t-norm and $\mathcal{S}\mathfrak{f}_{\mathscr{p}}$: $'\mathbb{X}'\mathbb{K}\wp \times '\mathbb{X}'\mathbb{K}\wp \times [0,\infty)\wp \rightarrow [0,1]\wp$ be a mapping. Considering $\mathfrak{f}_{\mathscr{p}}$ be a fuzzy partial metric space & $'\mathbb{X}'\mathbb{K}\wp$ is a soft metric over $\wp$. If specified conditions are satisfied $\forall \mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, \mathscr{r}_k^{p} \in \mathbb{K}\wp$ and $u, v \geq 0$, then triplet $(\mathbb{K}\wp, \mathcal{S}\mathfrak{f}_{\mathscr{p}}, \Theta)$ is said to be a soft fuzzy partial metric space:

i) '$\mathcal{S}\mathfrak{f}_{\mathscr{p}}(\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, 0) = 0$'

ii) '$\mathcal{S}\mathfrak{f}_{\mathscr{p}}(\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, u) = \mathcal{S}\mathfrak{f}_{\mathscr{p}}(\mathscr{q}_j^{\rho}, \mathscr{p}_i^{\rho}, u)$'

iii) '$\mathcal{S}\mathfrak{f}_{\mathscr{p}}(\mathscr{p}_i^{\rho}, \mathscr{r}_k^{p}, u + v) \geq \mathcal{S}\mathfrak{f}_{\mathscr{p}}(\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, u) \odot \mathcal{S}\mathfrak{f}_{\mathscr{p}}(\mathscr{q}_j^{\rho}, \mathscr{r}_k^{p}, v)$'

iv) ''$\mathcal{S}\mathfrak{f}_{\mathscr{p}}\ (\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, u) \leq 1, u > 0$'&'$\mathcal{S}\mathfrak{f}_{\mathscr{p}}(\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, u) = 1$' if f' $p(\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}) = 0$'

v) '$\mathcal{S}\mathfrak{f}_{\mathscr{p}}\ (\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, \odot)$: $(0,\infty)\wp \rightarrow [0,1]\wp$ is continuous,

Here '$\mathcal{S}\mathfrak{f}_{\mathscr{p}}(\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, u) = \frac{u}{u + p(\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho})}$'. If ('X'$\wp, \mathcal{S}\mathfrak{f}_{\mathscr{p}}, \Theta$), is a soft fuzzy partial metric space, then '$\mathfrak{f}_{\mathscr{p}}'$ is a fuzzy partial metric on $\mathbb{K}\wp$.

3.1.1. Example 3.1.1

Consider $(\mathbb{K}\wp,\ p)$ is a soft fuzzy partial metric space & $\mathscr{p}\Theta\mathscr{q} = \min\{\mathscr{p}, \mathscr{q}\}$ and $\mathscr{p}\Theta\mathscr{q} = \mathscr{p} \otimes \mathscr{q}$ are defined in $(\mathbb{K}\wp, p)$. Define mapping $\mathcal{S}\mathfrak{f}_{\mathscr{p}}$: $'\mathbb{X}'\mathbb{K}\wp \times '\mathbb{X}'\mathbb{K}\wp \times [0,\infty)\wp \rightarrow [0,1]\wp$ as:

$$\mathcal{S}\mathfrak{f}_{\mathscr{p}}(\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, u) = \frac{u}{u \oplus p(\mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho})}, \mathscr{p}_i^{\rho}, \mathscr{q}_j^{\rho}, \in \mathbb{K}\wp \text{ and } u \geq 0$$

3.1.2. Example 3.1.2

Universal set: $U = \{a, b, c\}$ = set of objects. Define $\mathfrak{f}_{\mathscr{p}}(x, y)$ is a fuzzy partial metric on U, here:

$$\mathfrak{f}_{\mathscr{p}}(a, b) = 0.8, \mathfrak{f}_{\mathscr{p}}(a, c) = 0.6, \mathfrak{f}_{\mathscr{p}}(b, c) = 0.7$$

Define $(F, \wp)$ is a soft set over U, here: $\wp$={parameter1, parameter2}

$$F(parameter1) = \{a(0.9), b(0.8)\}\ F(parameter2) = \{a(0.7), c(0.9)\}$$

We can combine the soft set $(F, \wp)$ and fuzzy partial metric $\mathfrak{f}_{\mathscr{p}}$ to define a soft fuzzy partial metric space.
Let's calculate the similarity between objects a and b under parameter1:

$\not{f}_{\wp}(a, b) = 0.8$ (fuzzy partial metric)

$F(parameter1)(a) = 0.9$ (soft set) $F(parameter1)(b) = 0.8$ (soft set)

The similarity between a and b under parameter1 can be calculated as

$$\begin{aligned} \mathcal{S}\not{f}_{\wp}(a, b) &= \not{f}_{\wp}(a, b) \times min\{F(parameter1)(a), F(parameter1)(b)\} \\ &= 0.8 \times min\{0.9, 0.8\} \\ &= 0.8 \times 0.8 = 0.64 \end{aligned}$$

This example demonstrates how soft sets can be used to extend fuzzy partial metric spaces and provide a more flexible and robust framework for modelling complex systems.

3.2. Definition 3.2

Every soft sequence $\{\mathcal{p}_i^{\rho m}\}$ in soft fuzzy partial metric space $(\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \Theta)$, is convergent to a soft point $\mathcal{q}_j^{\rho} \in \mathbb{K}\wp$,

if $\lim_{\wp\to\infty} \mathcal{S}\not{f}_{\wp}(\mathcal{p}_i^{\rho m}, \mathcal{q}_j^{\rho}, u) = 1, \forall\ u > 0.$ i.e. $p(\mathcal{p}_i^{\rho m}, \mathcal{q}_j^{\rho}) = 0$

Similarly, for any $u > 0, \varepsilon > 0\ \exists\ \mathcal{N}_0 \in \mathcal{Z}^+$ such that $\mathcal{S}\not{f}_{\wp}(\mathcal{p}_i^{\rho m}, \mathcal{q}_j^{\rho}, u) > 1 \odot \varepsilon, \forall\ m \geq \mathcal{N}_0$

3.3. Definition 3.3

Every soft sequence $\{\mathcal{p}_i^{\rho m}\}$ in soft fuzzy partial metric space $(\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \Theta)$, is Cauchy sequence in soft fuzzy partial metric space, if $\lim_{\wp\to\infty} \mathcal{S}\not{f}_{\wp}(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho n}, u) = 1, \forall\ u > 0.$ i.e. $p(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho n}) = 0.$
Similarly, for any $u > 0, \varepsilon > 0\ \exists\ \mathcal{N}_0 \in \mathcal{Z}^+$ such that $\mathcal{S}\not{f}_{\wp}(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho n}, u) > 1 \odot \varepsilon, \forall\ m, n\ \geq \mathcal{N}_0$

Remark, by definition 3.2, 3.3 conclude that:
i) If every Cauchy sequence in soft fuzzy partial metric space is convergent, then soft fuzzy partial metric space $(\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \Theta)$, is complete.
ii) If every soft fuzzy sequence in soft fuzzy partial metric space admits at least one convergent soft subsequence, then soft fuzzy partial metric space $(\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \Theta)$ is compact.

3.4. Definition 3.4

Consider $(\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \Theta)$ is a soft fuzzy partial metric space. Soft mapping $(\mathfrak{U}, \Omega): (\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \odot) \to (\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \odot)$ is a soft fuzzy contraction mapping on soft fuzzy partial metric space, if $\exists$ a soft real number $\alpha \in [0, 1]$ satisfying the condition.

$$\mathcal{S}\not{f}_{\wp}\left((\mathfrak{U}, \Omega)\mathcal{p}_i^{\rho}, (\mathfrak{U}, \Omega)\mathcal{q}_j^{\rho}, u\right) \geq \mathcal{S}\not{f}_{\wp}\left(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho}, \frac{u}{\alpha}\right), \forall \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp\ \&\ u > 0$$

3.5. Definition 3.5

The map $\Psi: \mathcal{R}(\wp) \to [0, \infty)\wp$ is a ψ – function which is subsequent conditions:
i) $\Psi(u) = 0 \Leftrightarrow u = 0$.
ii) Ψ is a non-decreasing function.
iii) Ψ is left continuous for $u > 0$.
iv) Ψ is continuous at $u = 0$.
v) $\Psi(u) \to \infty\ as\ u \to \infty$.

3.6. Definition 3.6

Consider $(\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \Theta)$ is a soft fuzzy partial metric space. Soft mapping $(\mathfrak{U}, \Omega): (\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \odot) \to (\mathbb{K}\wp, \mathcal{S}\not{f}_{\wp}, \odot)$ is said to be a $\Psi -$ contraction mapping on a soft fuzzy partial metric space, if $\exists$ a soft real number $\alpha \in [0, 1]$ satisfying the condition.

$$\mathcal{S}\not{f}_{\wp}\left((\mathfrak{U}, \Omega)\mathcal{p}_i^{\rho}, (\mathfrak{U}, \Omega)\mathcal{q}_j^{\rho}, \Psi(u)\right) \geq \mathcal{S}\not{f}_{\wp}\left(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho}, \Psi\left(\frac{u}{\alpha}\right)\right),$$

$\forall \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp\ \&\ u > 0$, here Ψ is a ψ – function.

4. RESULTS AND DISCUSSION

4.1. Theorem 4.1

Considering ($\mathbb{K}\wp$, $\mathcal{S}\mathfrak{f}_{\mathcal{p}}$, Θ) is a complete soft fuzzy partial metric space such that $\lim_{\wp\to\infty} \mathcal{S}\mathfrak{f}_{\mathcal{p}}(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho}, u) = 1,\ \forall\, \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp$. Then soft fuzzy partial contraction mapping $(\mathfrak{U},\Omega)\ on\ \mathbb{K}\wp$ admitted a common soft fixed point.

Proof: considering a soft point $\mathcal{p}_i^{\rho 0} \in \mathbb{K}\wp$ and construct a soft sequence $\{\mathcal{p}_i^{\rho m}\}$ such that $\mathcal{p}_i^{\rho m} = (\mathfrak{U},\Omega)^m \mathcal{p}_i^{\rho 0}$. By using induction, we get:

$$\mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho(m+1)}, u\right) \geq \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho 0}, \mathcal{p}_i^{\rho 1}, (\tfrac{u}{\propto^m})\right)$$

By above condition and property 3 of definition of soft fuzzy partial metric space, for any $\mathfrak{d} \in \mathbb{Z}^+$, we get:

$$\mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho(m+\mathfrak{d})}, u\right) \geq \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho(m+1)}, \frac{u}{\mathfrak{d}}\right) \underbrace{\odot \dots \dots \odot}_{\mathfrak{d}-times} \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho(m+\mathfrak{d}-1)}, \mathcal{p}_i^{\rho(m+\mathfrak{d})}, \frac{u}{\mathfrak{d}}\right)$$

$$\geq \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho 0}, \mathcal{p}_i^{\rho 1}, (\frac{u}{\mathfrak{d} \propto^m})\right) \underbrace{\odot \dots \dots \odot}_{\mathfrak{d}-times} \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho 0}, \mathcal{p}_i^{\rho 1}, (\frac{u}{\mathfrak{d} \propto^{m+\mathfrak{d}-1}})\right)$$

But given that $\lim_{\wp\to\infty} \mathcal{S}\mathfrak{f}_{\mathcal{p}}(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho}, u) = 1,\ \forall\, \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp$.

$$\mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho(m+\mathfrak{d})}, u\right) \geq \underbrace{1 \odot 1 \odot \dots \dots \odot 1}_{\mathfrak{d}-times} = 1$$

Hence, the soft fuzzy partial sequence $\{\mathcal{p}_i^{\rho m}\}$ is a Cauchy in ($\mathbb{K}\wp$, $\mathcal{S}\mathfrak{f}_{\mathcal{p}}$, Θ). Thus, it is convergent. Therefore, ($\mathbb{K}\wp$, $\mathcal{S}\mathfrak{f}_{\mathcal{p}}$, Θ) is complete. We obtain,

$$\{\mathcal{p}_i^{\rho m}\} \to \mathcal{q}_j^{\rho},\ \forall\, \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp$$

$$\lim_{\wp\to\infty} \mathcal{S}\mathfrak{f}_{\mathcal{p}}(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho m}, u) = 1, \qquad \forall\, \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp.$$

Then,

$$\mathcal{S}\mathfrak{f}_{\mathcal{p}}\left((\mathfrak{U},\Omega)\mathcal{q}_j^{\rho}, \mathcal{q}_j^{\rho}, u\right) \geq \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left((\mathfrak{U},\Omega)\mathcal{q}_j^{\rho}, (\mathfrak{U},\Omega)\mathcal{p}_i^{\rho m}, \tfrac{u}{2}\right) \odot \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left((\mathfrak{U},\Omega)\mathcal{p}_i^{\rho m}, \mathcal{q}_j^{\rho}, \tfrac{u}{2}\right)$$

$$\geq \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{q}_j^{\rho}, \mathcal{p}_i^{\rho m}, \tfrac{u}{2\propto}\right) \odot \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho(m+1)}, \mathcal{q}_j^{\rho}, \tfrac{u}{2}\right)$$

$$\geq 1 \odot 1 = 1$$

$$\mathcal{S}\mathfrak{f}_{\mathcal{p}}\left((\mathfrak{U},\Omega)\mathcal{q}_j^{\rho}, \mathcal{q}_j^{\rho}, u\right) = 1$$

Hence $(\mathfrak{U},\Omega)\mathcal{q}_j^{\rho} = \mathcal{q}_j^{\rho}$. Thus, $\mathcal{q}_j^{\rho}$ is a soft fixed point of $(\mathfrak{U},\Omega)$. It is simple to confirm that a soft fixed point of the soft fuzzy partial contraction mapping $(\mathfrak{U},\Omega)$ is unique and complete.

4.2. Theorem 4.2

Consider ($\mathbb{K}\wp$,$\mathcal{S}\mathfrak{f}_{\mathcal{p}}$,$\Theta$) is a complete soft fuzzy partial metric space such that $\lim_{\wp\to\infty} \mathcal{S}\mathfrak{f}_{\mathcal{p}}(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho}, u) = 1, \forall\, \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp$. Then $\Psi-$ contraction mapping $(\mathfrak{U},\Omega): (\mathbb{K}\wp, \mathcal{S}\mathfrak{f}_{\mathcal{p}}, \odot) \to (\mathbb{K}\wp, \mathcal{S}\mathfrak{f}_{\mathcal{p}}, \odot) on\ \mathbb{K}\wp$ admitted a common soft fixed point.

Proof: consider a soft point $\mathcal{p}_i^{\rho 0} \in \mathbb{K}\wp$ and construct a soft sequence $\{\mathcal{p}_i^{\rho m}\}$ such that $\mathcal{p}_i^{\rho m} = (\mathfrak{U},\Omega)^m \mathcal{p}_i^{\rho 0}$. By using induction, we get,

$$\mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho(m+1)}, u\right) \geq \mathcal{S}\mathfrak{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho 0}, \mathcal{p}_i^{\rho 1}, \Psi\,(\tfrac{v}{\propto^m})\right)$$

By above condition and property 3 of definition of soft fuzzy partial metric space, for any $\mathfrak{d} \in \mathbb{Z}^+$, we get:

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho(m+\mathfrak{d})}, u\right) \geq \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho(m+\mathfrak{d})}, \Psi(\mathrm{v})\right)$$
$$\geq \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho(m+1)}, \Psi\left(\frac{v}{\mathfrak{d}}\right)\right)\underbrace{\odot \ldots\ldots \odot}_{\mathfrak{d}-times}\mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho(m+\mathfrak{d}-1)}, \mathcal{p}_i^{\rho(m+\mathfrak{d})}, \Psi\left(\frac{v}{\mathfrak{d}}\right)\right)$$
$$\geq \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho 0}, \mathcal{p}_i^{\rho 1}, \Psi(\frac{v}{\mathfrak{d} \propto^m})\right)\underbrace{\odot \ldots\ldots \odot}_{\mathfrak{d}-times}\mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho 0}, \mathcal{p}_i^{\rho 1}, \Psi(\frac{v}{\mathfrak{d} \propto^{m+\mathfrak{d}-1}})\right)$$

But given that $\lim_{\wp\to\infty} \mathcal{S}\mathscr{f}_{\mathcal{p}}(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho}, u) = 1,\ \forall \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp$.

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m}, \mathcal{p}_i^{\rho(m+\mathfrak{d})}, u\right) \geq \underbrace{1 \odot 1 \odot \ldots\ldots \odot 1}_{\mathfrak{d}-times} = 1$$

Hence, the soft fuzzy partial sequence $\{\mathcal{p}_i^{\rho m}\}$ is a Cauchy in ($\mathbb{K}\wp$,$\mathcal{S}\mathscr{f}_{\mathcal{p}}$,$\Theta$). Thus, it is convergent. Therefore, ($\mathbb{K}\wp$,$\mathcal{S}\mathscr{f}_{\mathcal{p}}$,$\Theta$) is complete.
We obtain, $\{\mathcal{p}_i^{\rho m}\} \to \mathcal{q}_j^{\rho}, \forall \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp$

$$\lim_{\wp\to\infty} \mathcal{S}\mathscr{f}_{\mathcal{p}}(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho m}, u) = 1, \forall \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp.$$

Then,

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}\left((\mathfrak{U},\Omega)\mathcal{q}_j^{\rho}, \mathcal{q}_j^{\rho}, u\right) \geq \mathcal{S}\mathscr{f}_{\mathcal{p}}\left((\mathfrak{U},\Omega)\mathcal{q}_j^{\rho}, (\mathfrak{U},\Omega)\mathcal{p}_i^{\rho m}, \tfrac{u}{2}\right) \odot \mathcal{S}\mathscr{f}_{\mathcal{p}}\left((\mathfrak{U},\Omega)\mathcal{p}_i^{\rho m}, \mathcal{q}_j^{\rho}, \tfrac{u}{2}\right)$$
$$\geq \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{q}_j^{\rho}, \mathcal{p}_i^{\rho m}, \Psi(\tfrac{v}{2\propto})\right) \odot \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho(m+1)}, \mathcal{q}_j^{\rho}, \tfrac{v}{2}\right)$$
$$\geq 1 \odot 1 = 1$$
$$\mathcal{S}\mathscr{f}_{\mathcal{p}}\left((\mathfrak{U},\Omega)\mathcal{q}_j^{\rho}, \mathcal{q}_j^{\rho}, u\right) = 1$$

Hence $(\mathfrak{U},\Omega)\mathcal{q}_j^{\rho} = \mathcal{q}_j^{\rho}$. Thus, $\mathcal{q}_j^{\rho}$ is a "soft fixed point of $(\mathfrak{U},\Omega)$. The uniqueness of a soft fixed point of the soft fuzzy partial contraction mapping $(\mathfrak{U},\Omega)$is easily verified; it is complete.

4.3. Theorem 4.3

Consider ($\mathbb{K}\wp$,$\mathcal{S}\mathscr{f}_{\mathcal{p}}$,$\Theta$) is a complete soft fuzzy partial metric space such that $\lim_{\wp\to\infty} \mathcal{S}\mathscr{f}_{\mathcal{p}}(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho}, u) = 1, \forall \mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho} \in \mathbb{K}\wp$. Define continuous t-norm Θ as '$\mathcal{p}_i^{\rho}, \odot\, \mathcal{q}_j^{\rho}$=min $(\mathcal{p}_i^{\rho}, \mathcal{q}_j^{\rho})$'. Then $\Psi -$ contraction mapping $(\mathfrak{U},\Omega)$: $(\mathbb{K}\wp, \mathcal{S}\mathscr{f}_{\mathcal{p}}, \odot) \to (\mathbb{K}\wp, \mathcal{S}\mathscr{f}_{\mathcal{p}}, \odot)\, on\ \mathbb{K}\wp$ admitted a common soft fixed point.
Proof: consider a soft point $\mathcal{p}_i^{\rho 0} \in \mathbb{K}\wp$ and construct a soft sequence $\{\mathcal{p}_i^{\rho m}\}$ such that $\mathcal{p}_i^{\rho m} = (\mathfrak{U},\Omega)^m \mathcal{p}_i^{\rho 0}$. Assume that $\{\mathcal{p}_i^{\rho m}\}$ is not a Cauchy soft sequence. Then $\exists$ soft real numbers $u > 0, \varepsilon > 0$ satisfying that, $\exists\, m\ (\mathcal{N}_0), n\ (\mathcal{N}_0) \geq \mathcal{N}_0$ such that,

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho n(\mathcal{N}_0)}, u\right) > 1 \ominus \varepsilon, \forall\, \mathcal{N}_0 \in \mathcal{Z}^+$$

Choose $m(\mathcal{N}_0) < n(\mathcal{N}_0)$ such that $n\ (\mathcal{N}_0)$ is the lowest positive integer w. r. to $m\ (\mathcal{N}_0)$ that is satisfies above condition. Hence $\exists$ soft real numbers $u > 0, \varepsilon > 0$ for which two increasing sequences $\{m\ (\mathcal{N}_0)\}\ and\ \{n\ (\mathcal{N}_0)\}$, $m(\mathcal{N}_0) < n\ (\mathcal{N}_0)$ which satisfies (1) and (2).

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho(n(\mathcal{N}_0)-1)}, u\right) \leq 1 \ominus \varepsilon \quad (1)$$

$$\&\ \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho n(\mathcal{N}_0)}, u\right) > 1 \ominus \varepsilon \quad (2)$$

Finding a soft point $\mathcal{p}_i^{\rho n(\mathcal{N}_0)}$is necessary for the creation of such sequences, such that,

$\mathcal{p}_i^{\rho n(\mathcal{N}_0)} \notin \{\mathcal{q}_j^{\rho} / \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{q}_j^{\rho}, u\right) \leq 1 \ominus \varepsilon\}$ and

$\mathcal{p}_i^{\rho(n(\mathcal{N}_0)-1)} \in \{\mathcal{q}_j^{\rho} / \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{q}_j^{\rho}, u\right) \leq 1 \ominus \varepsilon\}$.

Creation of such sequence is possible because $\{ \mathcal{p}_i^{\rho m} \}$ is not a Cauchy soft sequence,

Since $q_j^{\rho} \in \mathbb{K}\wp, u_2 > u_1 > 0, \varepsilon > 0,$

$$\{ q_j^{\rho} / \mathcal{S}\mathscr{f}_{\mathcal{p}}(\mathcal{p}_i^{\rho}, q_j^{\rho}, u_1) \leq 1 \ominus \varepsilon\} \subset \{ q_j^{\rho} / \mathcal{S}\mathscr{f}_{\mathcal{p}}(\mathcal{p}_i^{\rho}, q_j^{\rho}, u_2) \leq 1 \ominus \varepsilon\}$$

It means that such sequence formation is attainable for $u > 0, \varepsilon > 0$, the construction of $\{\mathcal{p}_i^{\rho m(\mathcal{N}_0)}\}$ & $\{\mathcal{p}_i^{\rho n(\mathcal{N}_0)}\}$ satisfies condition (1) and (2) corresponding to any $g > 0, \varepsilon > 0$, where g $< u$. Now, Ψ is a ψ – function, for $u > 0$, $\exists\ g > 0$ such that $u > \Psi(g)$. Put= $\Psi(u_1)$, for some $u_1 > 0$ such that $\Psi(\frac{u_1}{\propto}) > \Psi(u_1)$, this is possible by using definition 3.7, condition (1) and (4). By condition (1) and (2), we get (3) to (5).

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho(n(\mathcal{N}_0)-1)}, \Psi(u_1)\right) \leq 1 \ominus \varepsilon \quad (3)$$

And

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho(n(\mathcal{N}_0))}, \Psi(u_1)\right) > 1 \ominus \quad (4)$$

Therefore,

$$1 \ominus \varepsilon < \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho(n(\mathcal{N}_0))}, \Psi(u_1)\right) \leq \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho(m(\mathcal{N}_0)-1)}, \mathcal{p}_i^{\rho(m(\mathcal{N}_0)-1)}, \Psi\left(\frac{u_1}{\propto}\right)\right)$$

i.e. $1 \ominus \varepsilon < \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho(m(\mathcal{N}_0)-1)}, \mathcal{p}_i^{\rho(m(\mathcal{N}_0)-1)}, \Psi\left(\frac{u_1}{\propto}\right)\right)$

As $\Psi(\frac{u_1}{\propto}) > \Psi(u_1)$, choosing u as $u < \{\Psi(\frac{u_1}{\propto}) \ominus \Psi(u_1)\}$
Choose $\mathcal{N}_0$ large such that:

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho(m(\mathcal{N}_0)-1)}, u\right) \leq 1 \ominus \varepsilon_1, 0 < \varepsilon_1 < \varepsilon \quad (5)$$

From (3) to (5) we get $1 \ominus \varepsilon < \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho(n(\mathcal{N}_0)-1)}, \Psi\left(\frac{u_1}{\propto}\right)\right)$

$$\leq \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho(n(\mathcal{N}_0)-1)}, (\Psi\left(\frac{u_1}{\propto}\right) \ominus u\right) \odot \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho(m(\mathcal{N}_0)-1)}, \mathcal{p}_i^{\rho(m(\mathcal{N}_0))}, u\right)$$

$$\leq \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho m(\mathcal{N}_0)}, \mathcal{p}_i^{\rho(n(\mathcal{N}_0)-1)}, \Psi(u_1)\right) \odot \mathcal{S}\mathscr{f}_{\mathcal{p}}\left(\mathcal{p}_i^{\rho(m(\mathcal{N}_0)-1)}, \mathcal{p}_i^{\rho(m(\mathcal{N}_0))}, u\right)$$

$$\leq (1 \ominus \varepsilon) \odot (1 \ominus \varepsilon_1$$

As $\varepsilon_1 < \varepsilon$, we have $(1 \ominus \varepsilon) < (1 \ominus \varepsilon_1)$ which is a contradiction. Thus $\{ \mathcal{p}_i^{\rho m} \}$ is a Cauchy soft sequence.

It is easy to confirm that a soft fixed point of the soft fuzzy partial contraction mapping $(\mathfrak{U}, \Omega)$ is unique and complete. It is easy to confirm that a soft fixed point of the soft fuzzy partial contraction mapping is unique. Now, we explain the example depends on Theorem 4.3.

4.4. Example 4.4

Considering set $\mathbb{K} = \{0.6, 0.7, 0.8\}$ & parameter set $\wp = \{1, 2\}$ with a continuous t-norm definite as $\mathcal{p}_i^{\rho} \odot q_j^{\rho}$=min $(\mathcal{p}_i^{\rho}, q_j^{\rho})$,$\forall \mathcal{p}_i^{\rho}, q_j^{\rho} \in$ [0, 1]$\wp$. Then $'\mathbb{X}'\wp\ (\mathbb{K}\wp) = \{0.6_1, 0.6_2, 0.7_1, 0.7_2, 0.8_1, 0.8_2\}$. We define $\mathcal{S}\mathscr{f}_{\mathcal{p}}$:'$\mathbb{X}'\wp \times$ '$\mathbb{X}'\wp \times (0, \infty)\wp \rightarrow [0, 1]\wp$ as follows: for all i, j$\in \wp$

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}(0.6_i, 0.7_j, u) = \mathcal{S}\mathscr{f}_{\mathcal{p}}(0.7_j, 0.6_i, u) = \begin{cases} 0, u = 0 \\ 0.9, 0 \leq u \leq 3 \\ 1, u > 3 \end{cases}$$

$$\mathcal{S}\mathscr{f}_{\mathcal{p}}(0.6_i, 0.8_j, u) = \mathcal{S}\mathscr{f}_{\mathcal{p}}(0.8_j, 0.6_i, u) = \mathcal{S}\mathscr{f}_{\mathcal{p}}(0.8_i, 0.7_j, u)$$
$$= \mathcal{S}\mathscr{f}_{\mathcal{p}}(0.7_j, 0.8_i, u) = \begin{cases} 0, u = 0 \\ 0.6, 0 \leq u \leq 8 \\ 1, u > 8 \end{cases}$$

$\mathcal{S}\mathfrak{f}_{\wp}(\wp_i^{\rho}, q_j^{\rho}, u) = 1 \Leftrightarrow \wp_i^{\rho} = q_j^{\rho},\ \wp_i^{\rho}, q_j^{\rho} \in \mathbb{K}\wp, u > 0$. Then $(\mathbb{K}\wp, \mathcal{S}\mathfrak{f}_{\wp}, \odot)$ is a complete soft fuzzy partial metric space. Consider $(\mathfrak{U}, \Omega)$ on $\mathbb{K}\wp$: $(\mathbb{K}\wp, \mathcal{S}\mathfrak{f}_{\wp}, \odot) \rightarrow (\mathbb{K}\wp, \mathcal{S}\mathfrak{f}_{\wp}, \odot)$ on $\mathbb{K}\wp$ as:

$$(\mathfrak{U}, \Omega)(0.6_1) = 0.7_1,\ (\mathfrak{U}, \Omega)(0.6_2) = 0.7_1$$
$$(\mathfrak{U}, \Omega)(0.7_1) = 0.7_1,\ (\mathfrak{U}, \Omega)(0.7_2) = 0.6_2$$
$$(\mathfrak{U}, \Omega)(0.8_1) = 0.8_2,\ (\mathfrak{U}, \Omega)(0.8_2) = 0.8_1$$

$\Psi(u) = u^{0.3}$. Then $\Psi -$contraction mapping $(\mathfrak{U}, \Omega)$ follows the requirements for theorem 3, we get a common fixed point 0.7_1.

5. CONCLUSION

We have investigated the fundamental concepts and properties of soft fuzzy partial metric spaces and developed significant fixed-point results by integrating soft set theory, fuzzy sets, and partial metrics. The given fixed-point theorems extend classical results to the context of soft fuzzy partial metrics by ensuring the existence and uniqueness of solutions to the soft fuzzy contraction mapping and $\Psi -$contraction under conditions appropriate to the relevant examples. Applications of soft fuzzy partial metric spaces in engineering, particularly in image processing and analysis, allow images to be represented as fuzzy sets where each pixel has a membership value indicating the degree of connection to a particular region or feature. Thus, soft fuzzy partial metric spaces can be used to measure similarity between images; for example, a set of images of different road types, such as highways, urban areas, and rural areas, can be represented as a soft fuzzy set, with each pixel indicating the degree of membership to a particular road type. A soft fuzzy partial metric space can be defined on a set of images, where the distance between images is a similarity measure based on the difference in membership values of corresponding pixels. In the future, the scope of research includes integration with other mathematical structures to develop hybrid fixed-point theorems, application of fuzzy fixed-point theory in S-metric spaces to address challenges of navigation and control systems, thereby improving stability and performance, and investigation of paired fixed-point theorems in fuzzy metric spaces that satisfy $\varphi -$contractive conditions to deepen understanding of interactions between mappings.

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Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Rohini R. Gore	✓	✓		✓	✓	✓			✓	✓				
Renu P. Pathak	✓	✓		✓	✓				✓	✓				

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

The authors confirm that the data supporting the findings of this study are available within the article.

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