Enhancing fraction multiplication and division performance through intelligent tutoring system with visual models

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ABSTRACT

Mathematical proficiency, especially fractions, help humans to succeed in their future. However, students tend to struggle in understanding fractions, one of them is natural number bias (NNB). This study aims to address this challenge by developing a web-based math intelligent tutoring system (ITS) using the cognitive tutor authoring tools (CTATs) to create a cognitive tutor with adaptive feedback and visual models to support students in learning fraction multiplication and division. This study compares the students' learning performance from two schools which the first school has aligned visual method with the system proposed. The results show that the experimental groups in the first school outperformed the control group with an average learning gain of 33.78%, while in the second school the control group outperformed the experimental group with average learning gain of 65.79%. Each school suggests statistically significant differences in students' results. This highlights the effectiveness of ITS depends not only on system features but also the importance of alignment between students' prior visual models and the visualization method in the system. Students with aligned visual method suggest positive feedback from the survey conducted, indicating its potential for improving fraction learning when it is integrated consistently with existing visual instruction.

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1. INTRODUCTION

Mathematical understanding and skills during school help humans to succeed in life aspects such as financial and academic [1]–[3]. It is important to understand and have good basic mathematical skills like arithmetic operations on whole numbers or fractions for our daily life tasks as we grow up. Solving fraction problems requires a good understanding in arithmetic operations, as it requires students to develop both their conceptual understanding and procedural skills. Conceptually, students are required to compare and order fractions based on their values, while procedurally, they must perform arithmetic operations involving fractions, including addition, subtraction, multiplication, and division [4], [5]. However, students still face challenges when learning fractions, even though they have already learned and understood whole numbers [6]. One of the most serious challenges is natural number bias (NNB) [7]. NNB refers to a tendency of students to use natural number concepts to interpret fractions and rational numbers [6]–[8].

As learning becomes easier and more reachable with the advancement of technology, research in intelligent tutoring system (ITS) has raised interest. ITS is a computer program that can take various forms and are designed to combine artificial intelligence (AI) methods to provide tutors that know what materials they are teaching, who they are teaching, and how to teach them [9]. It could adapt the students' ability and knowledge level through its four main components to select and recommend learning content, provide guidance and assist students, and provide private tutoring [10], [11]. The four main components of the ITS in general consists of knowledge module, tutor module, student module, and a user interface [9], [12]. In supporting ITS authoring and creating the ITS, there are various authoring tools such as cognitive tutor authoring tools (CTAT), autotutor script authoring tools (ASAT), and generalized intelligent framework for tutoring (GIFT) authoring tool (GAT) [13], [14]. CTAT is one of the authoring tools that enables the developers to create a cognitive tutor, or an example tracing tutor and ease them to create the system through the HTML editor to create user interface using the drag and drop features [10], [15]. To overcome the students' misconceptions, several studies suggest that utilizing representations through diagrams or models is an effective way and could improve students' learning performance, especially in learning fraction arithmetic traditionally [16], [17].

Currently, ITSs have been applied in various use cases. However, the development of the ITSs for math learning is still in its infancy since it only accounted for 15.4% (6 out of 39) of published ITS research [18]. Numerous researches in math ITS have been conducted, both with and without authoring tools to develop math ITSs rapidly like using CTAT. Previous research on ITSs for fraction multiplication and division word problems faced challenges in accounting for all possible patterns of student responses. Furthermore, researchers have utilized CTAT to develop mathematics ITSs more efficiently and rapidly. For instance, there were research that utilized CTAT to build ITSs for some mathematical tasks, such as solving filling slot problems [19], solving fraction arithmetic through example-tracing tutors [20], and ITS to practice fraction [21]. However, the filling slot problems ITS and ITS for-practice fraction did not include adaptive and immediate feedback for any kind of students' responses and the example-tracing tutors covers only several fraction word problems such as single-digit fraction addition and subtraction, and single-digit of whole number multiplication and division. The example-tracing tutor suggests that there were improvements in students learning through the ITS, but there were statistically no significant results between two conditions made [20].

A cognitive tutor is a method that utilizes cognitive models in a tutoring system [22], [23]. It interprets students' learning behavior using a cognitive model that represents the knowledge students are expected to acquire through production rules. Although developing a cognitive tutor is time-consuming because it requires AI programming, it offers greater flexibility and can recognize a wide range of student answers and solution strategies [23]. Studies that use CTAT and integrate representation diagrams to the system have suggested that these diagrams can help students better understand how to sort fractions [24].

Numerous efforts have been made by researchers to build math ITS, including development of math learning management system (LMS) through CTAT [25], math ITS for fraction multiplication and division without using authoring tools [18], math ITS for filling slot problems [19], math ITS to solve elementary equations for K-12 students [16], ITS for solving algebraic problems [26], and ITS to practice fraction [21]. Studies have shown that using CTAT to develop ITSs can reduce the complexity of the development process [19], [25]. Furthermore, several studies have proposed dialogue based ITS [16], [18] in order to improve students' engagement while studying through the system. These studies suggest that dialogue-based tutor could increase students' engagement with ITS. In the other hand, ITS to practice fraction support students in learning percentage concept, however the assignment in the ITS was 91% assigned by the teacher.

This study expects to fill the gap in previous studies, handle students' struggle in learning fraction by creating a cognitive tutor for learning fraction multiplication and division with adaptive feedback, visual model, and extending the type of fraction multiplication and division tutor in the system. Furthermore, this study aims to investigate and give insights on the effectiveness of the tutor through two schools which have the same and different methods of fraction modelling in the system. The proposed cognitive tutor provides step-by-step guidance for fraction multiplication and division problems, enabling student to learn fraction through a structured problem-solving.

The rest of this paper is organized as follows. Section 2 describes the tutor design, implementation, and instruments used to evaluate the system. Section 3 describes the results including the effect of visual model on both schools. Lastly, section 4 discusses conclusions.

2. METHOD

2.1. System overview

The system architecture proposed is the general ITS architecture, this study employed user interface, student model, tutor model, and domain model in the system. The system was developed using CTAT and deployed on a virtual private server (VPS) with Apache as the web server. This setup allows students to access the tutor through the web browser specifically Chrome in their computer or laptop online.

2.2. User interface

There are 4 user interfaces included in the system. The user interfaces were built through CTAT HTML editor which is one of CTAT's features that helps in rapid system development. It eases the user interface development through drag and drop features. In general, the user interface for each material has 4 main parts which are shown in Figure 1. Part 1 shows the problem defined by the system, the numbers are randomly generated in range one to ten; part 2 displays the model, the multiplication result, and the rewrite problem section; part 3 displays the section for student to fill in the simplified fractions; and finally part 4 displays a button to return to the homepage and choose another tutor, a button to refresh the page and generate a new question, a dialog box for the tutor to provide feedback and hints, and a "Done" button to submit the answer. To complete a session, students begin by rewriting the given problem to the designated input fields. Upon each submission on boxes, student fills the boxes, the system provides immediate feedback through a dialog box, indicating whether the student's response to the current step is correct or incorrect. If assistance is required, students may click the hint button to receive a contextual hint for the current step via the dialog box. Following the rewriting problem stage, the visualization is automatically triggered to support students conceptual understanding. Students are then required to enter the simplified fraction in the input fields at the bottom of the interface. Once all steps are answered correctly, the dialog box displays a completion message, prompting the student to proceed to a new question.

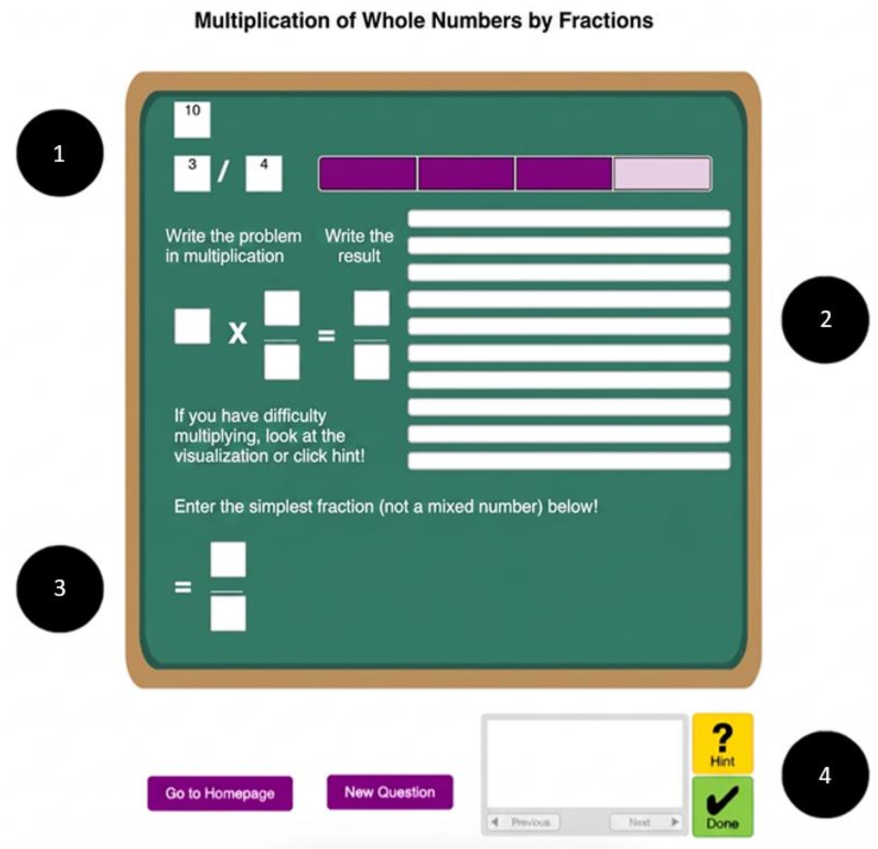


Figure 1. User interface in general

The tutor has four types of user interface for each material taught in the system. Figure 2 consecutively suggest the user interface for fraction multiplication and fraction division. Figure 2(a) suggests the user interface for whole number with fraction multiplication, while Figure 2(b) suggests the user interface for fraction with fraction multiplication. Moreover, Figures 2(c) and 2(d) consecutively suggest the user interface for whole number with fraction division and fraction with fraction division. All of the visualizations step and method are handled by the domain model. The CTAT components used in this user interface are CTAT fraction bar, CTAT text input, CTAT text area, CTAT table, and CTAT hint widget.

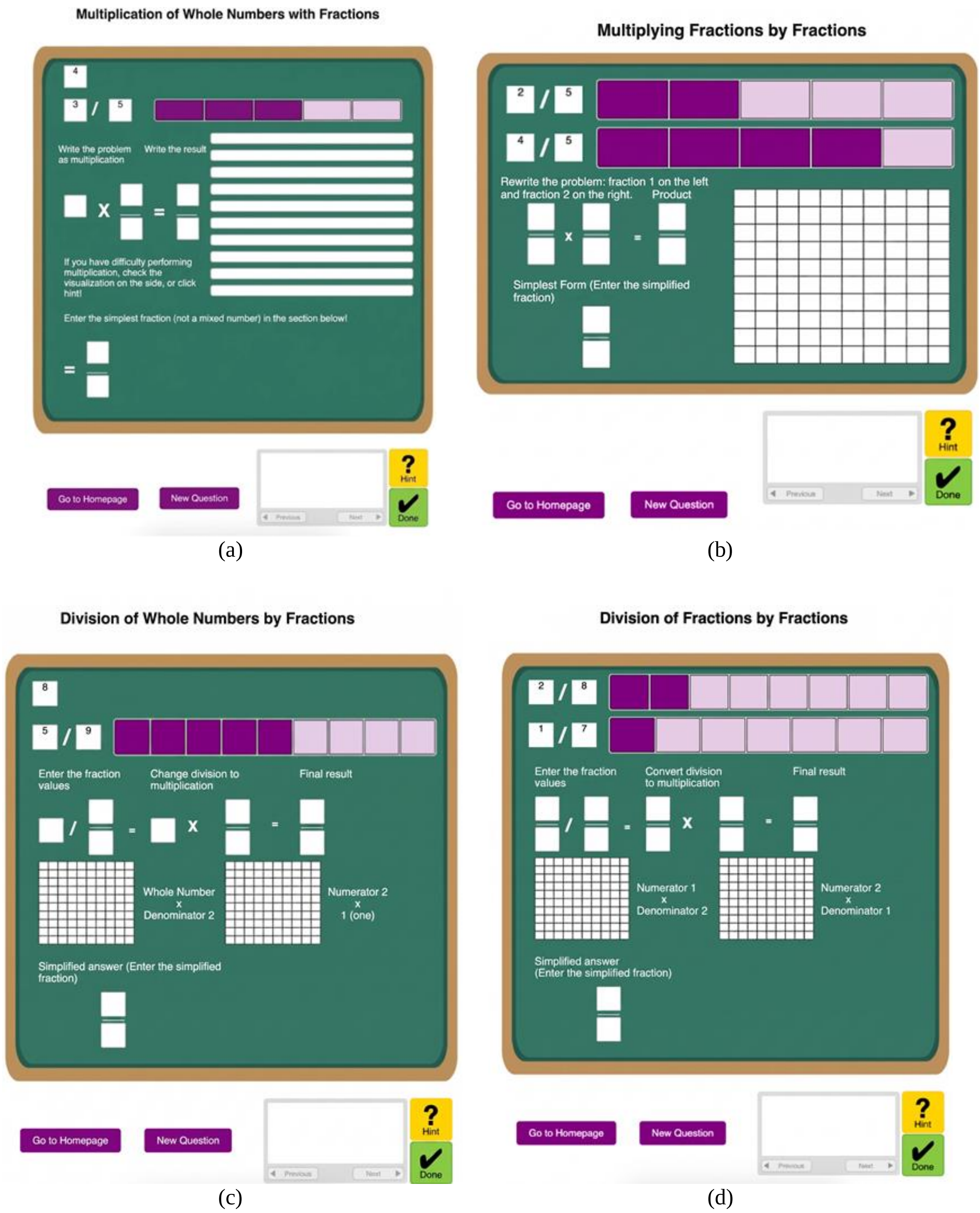


Figure 2. User interface (a) whole number with fraction multiplication, (b) fraction with fraction multiplication, (c) whole number with fraction division, and (d) fraction with fraction division

2.3. Student model

The student model loads students' historical information and knowledge representation while the student is working on the problem. Every student's response on each step will be recorded by the working memory so that the system can predict and provide adaptive feedback. For instance, if the student made an error, the student model will record how many errors the student already made and will be used as information for the tutor model to give appropriate feedback.

2.4. Tutor model

The tutor, in form of cognitive tutor, is responsible for providing any kind of feedback for students whether it's simple feedback, hints, or bug notifications. Figure 3 suggest the feedbacks included in the system to help students solving the problems. When a student correctly solves the current step, the tutor provides positive feedback through encouraging messages or praise to motivate the student to continue to the next step. For example, the tutor may display a message such as "Great! You're right! Next, let's calculate the result of fraction multiplication," as shown in Figure 3(a).

To overcome students' misconceptions and incorrect answers, two-level feedback for student misconceptions was designed in the tutor in the form of production rules as buggy rules. Figure 4 suggests how the tutoring cycle by the tutor works for each step the student is currently working on. Students will enter their answers in a specific step box, then the student's responses will be examined by the CTAT JavaScript tracer model to identify if they match to the correct rule or a buggy rule.

If an incorrect answer is submitted, the tutor delivers feedback through a dialog box designed to stimulate cognitive conflict and encourage reflection on the solution. Figure 3(b) suggest an example of feedback when the student first time made an incorrect answer. The feedback given tells the student that the student's answer is still incorrect and then it will provoke cognitive conflict in the form of reverse statement to remind them that simple fractions are whose numerators and denominators cannot be divided by numbers other than 1. Furthermore, when students consistently respond incorrectly to the current step, the tutor will provide the feedback in form of correct and final solution for the current step. For instance, during fraction simplification, if a student submits an answer that is not in its simplest form, the tutor identifies the misconception and provides initial feedback to indicate that further simplification is required. If the student still answers incorrectly, the second feedback provides a short message informing the student of the mistake and giving the correct solution for the step the student is working on as shown in Figure 3(c). The feedback tells the student the solution and where to fill their answer.

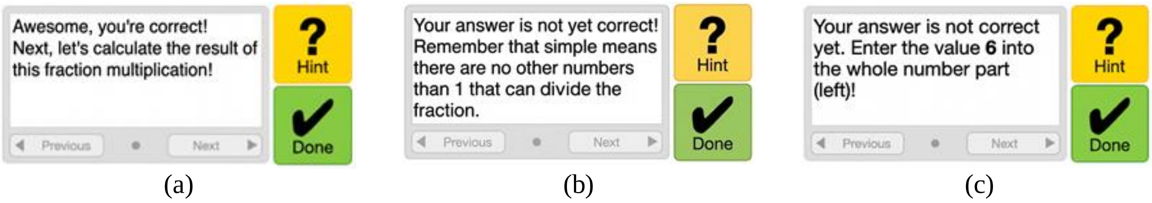


Figure 3. Feedback included in the system (a) positive feedback, (b) first error feedback, and (c) second error feedback (solution feedback)

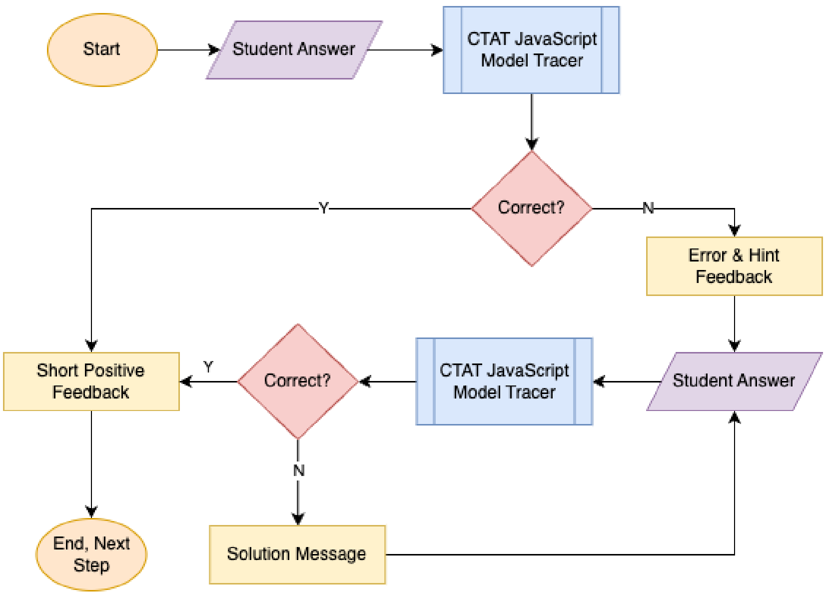


Figure 4. Tutoring flow

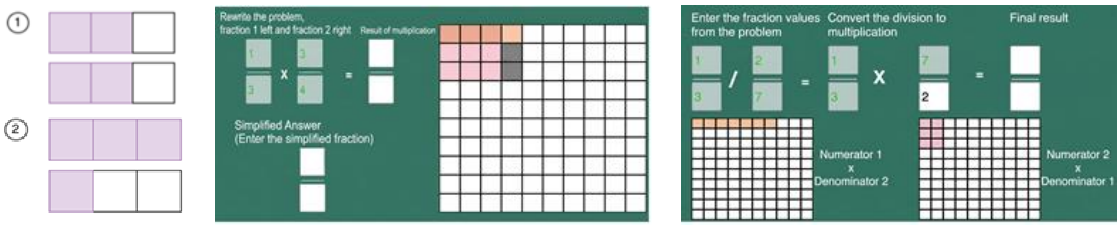
2.5. Domain model

The domain model includes working memory and the skills covered in the system. Each topic on fraction multiplication and division covers two skills. For fraction multiplication, students learn to multiply whole numbers by fractions and fractions by fractions. For fraction division, students learn to divide whole numbers by fractions and fractions by fractions. The questions provided are based on the syllabus of sixth grade mathematics textbooks in Indonesia. The ITS taught each skill on separate interfaces each with its own production rule and model. Figure 5 suggests the visual models included in the system.

Figure 5(a) suggests how the domain model represents the multiplication of whole number with fraction. For instance, the problem is $2 \times \frac{2}{3}$, the model will color 2 bars that equal to $\frac{2}{3}$, then each colored box will be moved to the fraction box above. Once a fraction bar is filled, the model continues placing the remaining shaded boxes in the fraction bar above until all shaded boxes have been transferred. To interpret the result, the numerator will be the total number of the colored boxes, while the denominator will be the number of boxes that can be loaded into a fraction bar.

Next, Figure 5(b) illustrates how the domain model represents the multiplication of fraction with fraction in the form of $\frac{a}{b} \times \frac{c}{d}$. The first column of the table will be colored as the initial point to illustrate $\frac{a}{b}$. Next, first row of the table will be colored as initial point to illustrate $\frac{c}{d}$. Both colors of the fractions will then overlap each other to show the numerator of the fraction, and the model will color the uncolored cells after the two fractions overlap to show that all colored cells show the denominator, which is $b \times d$.

Lastly, the domain model has the same way in representing both type of fraction division. The system uses two tables to represent the result of numerator and denominator. Figure 5(c) suggests the model representation for both type of fraction division included in the system. For instance, if we have fraction division in form of $\frac{a}{b} \div \frac{c}{d}$ and $a \div \frac{c}{d}$. The left table will color the row and column as $a \times d$, while the right table will color the row and column as $b \times c$ for fraction with fraction division or $1 \times c$ for whole number with fraction division.



(a) (b) (c)

Figure 5. Visual model of (a) whole number with fraction multiplication, (b) fraction with fraction multiplication, and (c) fraction with fraction and whole number with fraction division

2.6. Learning scenario

The learning scenario, as shown in Figure 6, starts with pre-test, followed by the learning phase, post-test, and usability and user experience survey. A pre-test will be conducted on students to assess their abilities before receiving material related to multiplication and division of fractions. Next, there will be a learning session to perform certain behaviors to students, which will be divided into two groups, which are control group and experimental group. The experimental group will be given learning instructions through the math ITS, accessible through a web browser, while the control group will receive learning instructions in a traditional classroom setting with their regular teacher. After the learning session is complete, students will be given a post-test to assess their learning outcomes, along with a usability and user experience survey to evaluate the effectiveness of the math ITS. The results of the pre-test, post-test, and survey will then be analyzed and evaluated.

The learning scenario lasted for 70 minutes for both the control and experimental group, consisting of 15 minutes pre-test, 40 minutes learning, and 15 minutes post-test. Observations were conducted on each student for 40 minutes of time spent learning through the math ITS. In the first school's experimental group, there were 3 students who only engaged with one of the four tutors provided, and during the 40 minutes of learning, these students spent their time accessing unrelated applications such as games or YouTube. Therefore, the data from these 3 students were excluded from the valid datasets, as they were considered not to have fully engaged with the math ITS during the allocated 40 minutes.

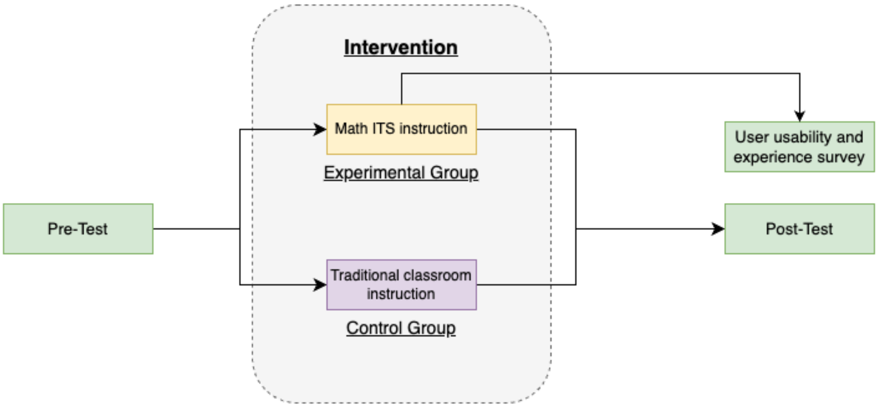


Figure 6. Students learning scenario

2.7. Assessments

The pre-test and post-test both contained the same set of 20 arithmetic multiple-choice questions. The assessment was constructed by the researcher, aligned with the sixth-grade mathematics syllabus mandated by the Indonesian national curriculum, and reviewed and validated by the classroom teacher. There are five questions representing each of the following topics: fraction with fraction multiplication, fraction with whole number multiplication, fraction with fraction division, and fraction with whole number division in the test. This test set will be utilized for comparisons to measure whether there were learning gain among students or not.

2.8. Usability and user experience survey

To evaluate usability and user experience, students in the experimental group were asked to complete a survey consisting of items rated on a 5-point Likert scale. The survey consisted of three parts which are system effectiveness survey, user experience survey on adaptive feedback features, and user experience survey related to visualization features provided in the tutor. Table 1 shows the items included in the usability and user experience surveys.

Table 1. Items of usability and user experience survey

No	Category	Items
1	System effectiveness	I am excited and satisfied studying with this ITS.
2	System effectiveness	Studying with this ITS makes learning fraction multiplication and division easier.
3	System effectiveness	I feel that I can understand the material better by studying with this ITS.
4	System effectiveness	I feel that this ITS has helped me learn multiplication and division of fractions well.
5	System effectiveness	I feel more confident learning with this ITS.
6	System effectiveness	I want to spend more time learning multiplication and division of fractions through this ITS.
7	Adaptive feedback	Feedback messages provided by the ITS when I answered questions or gave incorrect answers were clear and easy to understand.
8	Adaptive feedback	Feedback messages delivered by ITS made me feel like I was participating in a classroom learning experience with a teacher, even though it was only a system.
9	Visualization	The visualizations displayed by the system correspond to the lessons taught before taking the pre-test.
10	Visualization	The layout of images/visualizations and visual elements in the system are neat.
11	Visualization	The images/visualizations displayed by the system helped me understand how to solve fractions multiplication and division problems.

2.9. Participants

This study was conducted with 88 of 6th graders from two schools. There were 55 students from a school that aligned with the fraction multiplication and division model used in the system and there were 33 students, the rest of them, from the second school that utilized different visual model in learning fraction multiplication and division. There were also two qualified teachers from each school invited during the experiment. The teachers assisted in teaching, preparing the computer lab, and guiding students during experiment.

3. RESULTS AND DISCUSSION

3.1. Students learning performance

Students' average score and learning gain based on pre-test and post-test results as shown on Table 2. In (1) was used to calculate each student's learning gain [27]. The learning gain of each student is calculated based on the differences between the post-test score and pre-test score of the current students (L) divided by the student's pre-test score.

$$Learning\ gain\ = \left(\frac{Post\_Test\_Score_L - Pre\_Test\_Score_L}{Pre\_Test\_Score_L}\right) \times 100\% \quad (1)$$

Both the control and experimental groups in each school had almost the same pre-test average scores. This shows that both groups had approximately the same knowledge base in understanding multiplication and division of fractions. Highest average learning gain overall were achieved by the control group in the second school which is 65.79%. Since the second school were using different method of visualization with the math ITS proposed, experimental group struggled in understanding the visualization in the math ITS, therefore they performed less better than the control group. Meanwhile, in the first school, the experimental group perform better than the control group with a learning gain of 33.7%, which was higher than the control group that achieved average 13.25% of learning gain. This indicates that there is room to improve student learning performance for students by using an ITS.

Table 2. Students' results on both school

School	Group	N	Pre-test mean	Post-test mean	Learning gain mean (%)	Pre-test std.dev	Post-test std.dev
First school	Experimental	24	53.1250	65.2083	33.78	24.46	25.08
	Control	28	53.0357	56.9643	13.25	26.46	26.81
Second school	Experimental	15	24.3333	28.6667	31.30	7.76	11.25
	Control	18	26.3889	38.6111	65.79	11.98	11.47

Table 3 suggests comprehensive results through paired sample t-test of the experimental and control group of the first school. Experimental group suggests significant result with significance 0.001 ($p < 0.05$) outperforms the control group with significance of 0.077 ($p > 0.05$). This indicates that experimental group in the first school had good training through the math ITS. The adaptive feedback and visualizations model supports the students' fraction multiplication and division learning.

Table 3. Paired sample t-test results of first school

Comparison	Mean difference	Std.dev	Std. error mean	95% confidence interval of the difference		T-stats	df	sig. p-one side	sig. p-two sides
				Lower	Upper				
Pair 1: pre-test post-test experimental group	-12.083	16.998	3.469	-19.26	-4.90	-3.482	23	0.001	0.002
Pair 2: pre-test post-test control group	-3.928	14.165	2.677	-9.42	1.564	-1.468	27	0.077	0.154

Furthermore, Table 4 shows the comprehensive results through paired sample t-test of the experimental and control group of the second school. It suggests contrasting results of the first school, where the control group suggests significant results. This suggests students that switch to another visualization method struggled in their fraction multiplication and division learning in limited time.

Table 4. Paired sample t-test results of second school

Comparison	Mean difference	Std.dev	Std. error mean	95% confidence interval of the difference		T-stats	df	sig. p-one side	sig. p-two sides
				Lower	Upper				
Pair 1: post-test pre-test experimental group	4.333	13.610	3.514	5.942	18.502	4.106	14	0.119	0.238
Pair 2: post-test pre-test control group	12.222	12.628	2.976	-3.203	11.870	1.233	17	< 0.001	< 0.001

Table 5 shows independent sample t-test of each group's post-test in the first school. The Levene test for equality variances has significance 0.002 ($p < 0.05$) which suggest the equal variances between two groups are not assumed. The result suggests there is a statistically significant difference between the experimental group's and the control group's mean with significance of 0.044 ($p < 0.05$) with a mean difference of 14.375. This improvement aligns with recent findings of previous study which observed that math ITSs capable of identifying misconceptions in real-time allow students to significantly outperform those receiving traditional instruction [18]. Furthermore, the performance gains in this group mirror the results previous study on adaptive prompts which showed that personalized guidance leads to higher post-test scores and increased efficiency [20], [28].

Table 5. Independent sample t-test of first school

Comparison	Levene test for equality variances sig.	Mean diff.	df	t	95% confidence interval of the difference Lower	Upper	sig. p-one side	sig. p-two sides
Equal variances assumed	0.002	-14.375	50	-2.006	-28.765	0.015	0.025	0.050
Equal variances not assumed		-14.375	47.03	-2.070	-28.345	-0.040	0.022	0.044

Table 6 shows independent sample t-test of each group's post-test in the second school. The significance of the Levene test for equality variances is < 0.001 ($p < 0.05$) which suggest the independent samples t-test for equal variances were not assumed. The results suggest that there is statistically significant difference between the two groups' mean with the significance < 0.001 ($p < 0.05$) with a mean difference of 36.541.

Table 6. Independent sample t-test of second school

Comparison	Levene test for equality variances sig.	Mean diff.	df	t	95% confidence interval of the difference Lower	Upper	sig. p-one side	sig. p-two sides
Equal variances assumed	< 0.001	-36.541	37	-5.27	-50.581	-22.502	< 0.001	< 0.001
Equal variances not assumed		-36.541	34.27	-6.182	-48.550	-24.533	< 0.001	< 0.001

Based on these results, the first school which has the same visualization method as the math ITS, suggests good performance in student learning in experimental group outperformed the control group. In contrast, the second school, which used different methods of visualization in solving the fraction multiplication and division problem, suggests significantly lower performance in student learning in experimental group than the control group. The control group in the second school had good training with their teacher through their own visualizations make them outperformed the experimental group of the second school.

3.2. Usability and user experience survey results

Figure 7 presents the first school students' feedback on system effectiveness, feedback, and visualization included in the system. The numbers in the figure suggest the question number in the Table 1. Based on these results, around more than 60% (agree and strongly agree) students out of 24 students on each items asked are satisfied with the system as on questions 1 to 6 and they felt the system's feedback are helpful based on responses on question 7 and 8, and based on responses on questions 9 to 11, they agreed that the visualizations corresponded to their materials, were tidy, and helpful for them to solve the fraction multiplication and division problems.

Figure 8 shows the feedback from the second school students on system effectiveness, feedback, and visualization. In general, more than 60% of the 15 students (agree and strongly agree) felt the system was effective based on their responses to questions 1 to 4. However, for item number 5, about 13% of second school students disagreed that they felt confident when learning through the system proposed.

Furthermore, there were only 40% of students from second school (agree and strongly agree) wanted to spend more time studying with the system. This suggests that the second school students were more comfortable to studying directly with teacher with visualizations that their teacher have taught. These findings were also supported by the students' responses on items number 9 that the visualizations used were different, so in item number 7 fewer than 50% of students felt that the feedback helped them to solve the problems. Despite these limitations, most of the students found out that the visualizations finally help them to solve the problems based on their responses in item number 11.

Results on system effectiveness, feedback, and visualizations on both schools suggest that the students recognized the utility of the adaptive scaffolding and the interactive behavior of the tutor, even in an environment where learning gains were not as high as expected. This positive perception indicates that the system's interface and step-by-step guidance were accessible to the learners. It is also highlighted by prior study that real-time feedback in is essential for reducing student anxiety and maintaining motivation during difficult topics like fractions [29], [30]. Furthermore, the students results and students' feedback towards the system is supported by previous finding who demonstrated that integrating diverse visual modalities, such as images and tailored instructional sequences, significantly enhances students' comprehension of fractions and decimal numbers [29]. Previous study found that when digital learning environments prioritize the redesign of visual models to be more intuitive, students show marked improvements in performance compared to non-personalized methods. This aligns with this study's findings in the first school, where the alignment of the system's visualization with the students' prior mental models served as a crucial bridge for conceptual understanding. By providing a clear relational representation of fraction magnitudes, such visual tools effectively mitigate the cognitive load typically associated with abstract arithmetic operations.

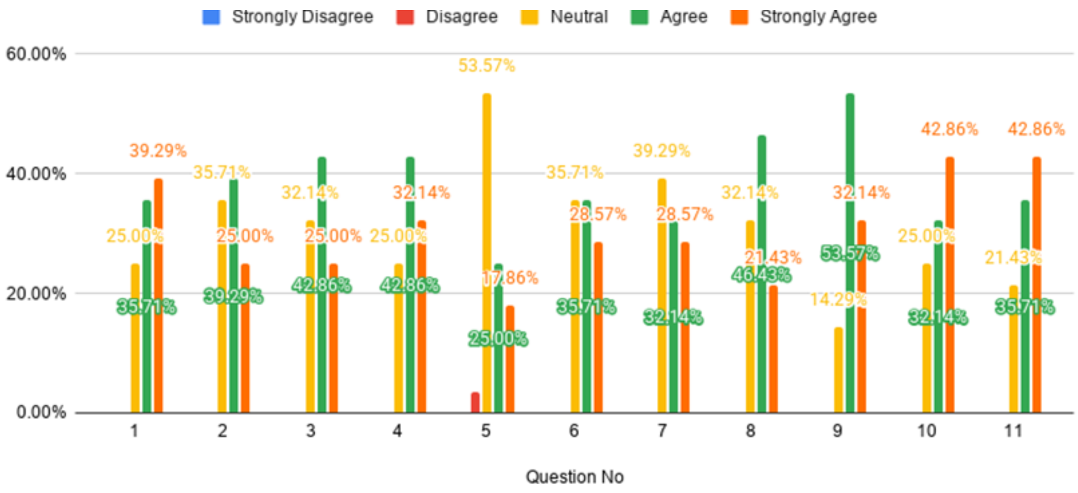


Figure 7. First school students' feedback on system effectiveness, feedback, and visualization

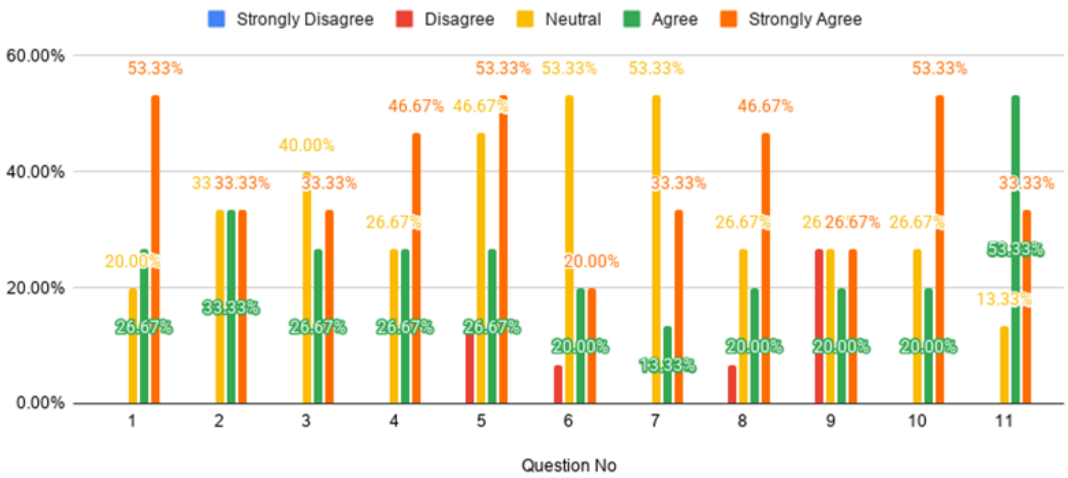


Figure 8. Second school students' feedback on system effectiveness, feedback, and visualization

3.3. Cost analysis
During this study, the system was developed using CTAT and deployed on a VPS running on Apache as web-server, allowing the students to access the tutor online through a web-browser without additional software installation. Both schools accessed the system with stable performance and no major technical issues. In the term of cost, the hosting cost was approximately IDR 25,000 per month for 100 users in the website and ".my.id" domain IDR 53,000 per year. This system and budgets indicating that the tutor is affordable to maintain and feasible for implementation.

4. CONCLUSION
The experimental results from two schools revealed that the effectiveness of the ITS strongly depends on the alignment of its visualization approach with students' prior learning experiences. In the first school, where the visualization in the system matched the teachers' method, the experimental group achieved a 33.78% average learning gain, compared with 13.25% for the control group, with a statistically significant difference between the groups ($p = 0.044$). In contrast, in the second school, where the visualization differed from the teacher's method, the experimental group achieved a 31.30% average learning gain, while the control group achieved 65.79%, with a statistically significant difference ($p < 0.001$). These findings highlight visual alignment as an important factor influencing the effectiveness of ITS, indicating that the success of adaptive feedback and visual models depends not only on the instructional features but also on the consistency between system-provided visual representations and students' existing learning experiences. Therefore, visualization design should be carefully considered when developing ITSs for mathematics learning. Further research with larger and more diverse student populations is needed to validate these findings and improve the flexibility of visual models for different learning contexts.

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C : **C**onceptualization
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D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT
Authors state no conflict of interest.

INFORMED CONSENT
We have obtained informed consent from all individuals included in this study. Specifically, written consent was collected from the parents of the students, and additional permission was granted by their

teachers before any data collection. All participants were informed about the purpose of the study, the procedures involved, and their right to withdraw at any stage.

DATA AVAILABILITY

The data that support the findings of this study are available from the first author, [IBIK], upon reasonable request.

REFERENCES

[1] D. T. Braak, R. Lenes, D. J. Purpura, S. A. Schmitt, and I. Størksen, "Why do early mathematics skills predict later mathematics and reading achievement? the role of executive function," *Journal of Experimental Child Psychology*, vol. 214, Feb. 2022, doi: 10.1016/j.jecp.2021.105306.

[2] H. Serin, "The significance of mathematical literacy in today's society," *International Journal of Social Sciences & Educational Studies*, vol. 10, no. 2, pp. 396–402, 2023, doi: 10.23918/ijsses.v10i2p396.

[3] D. W. Braithwaite, J. McMullen, and M. A. Hurst, "Cross-notation knowledge of fractions and decimals," *Journal of Experimental Child Psychology*, vol. 213, Jan. 2022, doi: 10.1016/j.jecp.2021.105210.

[4] M. W. H. Spitzer, L. Bardach, E. Richter, Y. Strittmatter, and K. Moeller, "A psychological network analysis to examine interdependencies between fraction and algebra subtopics in an intelligent tutoring system," *Journal of Computer Assisted Learning*, vol. 41, no. 4, Aug. 2025, doi: 10.1111/jcal.70093.

[5] L. İncikabı, P. Ayanoğlu, and R. Uysal, "Sixth-grade students' procedural and conceptual understandings of division operation in a real-life context," *lnternational Electronic Journal of Elementary Education*, vol. 13, no. 1, pp. 35–45, Oct. 2020, doi: 10.26822/iejee.2020.171.

[6] M. Hariyani, T. Herman, D. Suryadi, and S. Prabawanto, "Exploration of student learning obstacles in solving fraction problems in elementary school," *International Journal of Educational Methodology*, vol. 8, no. 3, pp. 505–515, Aug. 2022, doi: 10.12973/ijem.8.3.505.

[7] F. Reinhold, S. Hoch, B. Werner, J. R.-Gebert, and K. Reiss, "Learning fractions with and without educational technology: what matters for high-achieving and low-achieving students?," *Learning and Instruction*, vol. 65, Feb. 2020, doi: 10.1016/j.learninstruc.2019.101264.

[8] V. A. Avgerinou and A. Tolmie, "Inhibition and cognitive load in fractions and decimals," *British Journal of Educational Psychology*, vol. 90, no. S1, pp. 240–256, Jun. 2020, doi: 10.1111/bjep.12321.

[9] H. Nwana, "Intelligent tutoring systems: an overview," *Artificial Intelligence Review*, vol. 4, no. 4, pp. 251–277, 1990, doi: 10.1007/BF00168958.

[10] H. Wang *et al.*, "Examining the applications of intelligent tutoring systems in real educational contexts: a systematic literature review from the social experiment perspective," *Education and Information Technologies*, vol. 28, no. 7, pp. 9113–9148, Jul. 2023, doi: 10.1007/s10639-022-11555-x.

[11] O. Shvets, K. Murtazin, M. Meeter, and G. Piho, "Towards a domain model for learning and teaching," in *12th International Conference on Model-Based Software and Systems Engineering*, 2024, pp. 288–296, doi: 10.5220/0012471400003645.

[12] M. Pappas and A. Drigas, "Incorporation of artificial intelligence tutoring techniques in mathematics," *International Journal of Engineering Pedagogy*, vol. 6, no. 4, pp. 12–16, Nov. 2016, doi: 10.3991/ijep.v6i4.6063.

[13] A. Alkhatlan and J. Kalita, "Intelligent tutoring systems: a comprehensive historical survey with recent developments," *International Journal of Computer Applications*, vol. 181, no. 43, pp. 1–20, Mar. 2019, doi: 10.5120/ijca2019918451.

[14] K. Brawner, "Authoring tools for adaptive training – an overview of system types and taxonomy for classification," in *Foundations of Augmented Cognition*, Cham, Switzerland: Springer, 2015, pp. 562–569, doi: 10.1007/978-3-319-20816-9_54.

[15] V. Aleven, B. M. McLaren, J. Sewall, and K. R. Koedinger, "The cognitive tutor authoring tools (CTAT): preliminary evaluation of efficiency gains," in *Intelligent Tutoring Systems*, Berlin, Heidelberg: Springer, 2006, pp. 61–70, doi: 10.1007/11774303_7.

[16] A. M. M. Lopes, J. F. D. M. Netto, R. A. L. D. Souza, A. B. Mourao, T. O. Almeida, and D. P. R. D. Lima, "Improving students skills to solve elementary equations in k-12 programs using an intelligent tutoring system," in *2019 IEEE Frontiers in Education Conference (FIE)*, Oct. 2019, pp. 1–9, doi: 10.1109/FIE43999.2019.9028363.

[17] P. G. Sidney, J. F. Shirah, L. Zahrn, and C. A. Thompson, "Diagrams support spontaneous transfer across whole number and fraction concepts," *Contemporary Educational Psychology*, vol. 69, Apr. 2022, doi: 10.1016/j.cedpsych.2022.102066.

[18] S.-C. Shih, C.-C. Chang, B.-C. Kuo, and Y.-H. Huang, "Mathematics intelligent tutoring system for learning multiplication and division of fractions based on diagnostic teaching," *Education and Information Technologies*, vol. 28, no. 7, pp. 9189–9210, Jul. 2023, doi: 10.1007/s10639-022-11553-z.

[19] F. Hermawan, "Using intelligent tutoring systems through cognitive tutor authoring tools to solve filling slot problems," *Jurnal Pendidikan Matematika (Kudus)*, vol. 5, no. 1, pp. 93–102, Jun. 2022, doi: 10.21043/jpmk.v5i1.14484.

[20] S. M. D. L. Burr, H. Douglas, M. Vorobeva, and K. Muldner, "Refuting misconceptions: computer tutors for fraction arithmetic," *Journal of Numerical Cognition*, vol. 6, no. 3, pp. 355–377, Dec. 2020, doi: 10.5964/jnc.v6i3.310.

[21] M. W. H. Spitzer, M. R.-Garcia, and K. Moeller, "Basic mathematical skills and fraction understanding predict percentage understanding: evidence from an intelligent tutoring system," *British Journal of Educational Technology*, vol. 56, no. 3, pp. 1122–1147, May 2025, doi: 10.1111/bjet.13517.

[22] S. Ritter, J. R. Anderson, K. R. Koedinger, and A. Corbett, "Cognitive tutor: applied research in mathematics education," *Psychonomic Bulletin & Review*, vol. 14, no. 2, pp. 249–255, Apr. 2007, doi: 10.3758/BF03194060.

[23] V. Aleven, B. M. McLaren, J. Sewall, and K. R. Koedinger, "A new paradigm for intelligent tutoring systems: example-tracing tutors," *International Journal of Artificial Intelligence in Education*, vol. 19, no. 2, pp. 105–154, May 2009, doi: 10.3233/IRG-2009-19(2)02.

[24] M. A. Rau, V. Aleven, and N. Rummel, "Successful learning with multiple graphical representations and self-explanation prompts," *Journal of Educational Psychology*, vol. 107, no. 1, pp. 30–46, Feb. 2015, doi: 10.1037/a0037211.

[25] C. L. King, Vincent, Kelvin, H. L. H. S. Warnars, N. Nordin, and W. H. Utomo, "Intelligent tutoring system: learning math for 6th-grade primary school students," *Education Research International*, vol. 2021, pp. 1–10, Jun. 2021, doi: 10.1155/2021/5590470.

[26] P. A. Jaques *et al.*, "Rule-based expert systems to support step-by-step guidance in algebraic problem solving: the case of the tutor PAT2Math," *Expert Systems with Applications*, vol. 40, no. 14, pp. 5456–5465, Oct. 2013, doi: 10.1016/j.eswa.2013.04.004.
[27] N. Singh, V. K. Gunjan, and J. M. Zurada, *Cognitive tutor: custom-tailored pedagogical approach*. Singapore: Springer, 2022, doi: 10.1007/978-981-19-5197-8.
[28] H. Liu, Y. Zhang, and J. Jia, "The design of guiding and adaptive prompts for intelligent tutoring systems and its effect on students' mathematics learning," *IEEE Transactions on Learning Technologies*, vol. 17, pp. 1353–1363, 2024, doi: 10.1109/TLT.2024.3382000.
[29] Á. P. Loriente, J. A. G. -Calero, S. T. -Olivares, and J. D. O. -Muñoz, "Enhancing mathematics performance in primary education: the impact of personalized learning on fractions and decimal numbers," *Education and Information Technologies*, vol. 30, no. 11, pp. 15961–15991, Jul. 2025, doi: 10.1007/s10639-025-13428-5.

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