

# Method for the Solution of Interval Systems of Linear Equations

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## ABSTRACT

System of linear equation (crisp) can be solved in a various way .In the real world problem parameters are in interval form .In this paper we discuss the solution of interval system of linear equations and proposed a new method for handling this type of system of linear equations. In this model we consider the coefficient matrix and the right vector as interval. Example problems are given to have the efficiency and powerfulness of the proposed method.

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## 1. INTRODUCTION

System of linear equation can be solved by different methods. If the coefficient matrix and right vector are in crisp then this system can be solved by the Cramer's rule, Gauss-Elimination method, Gauss-Jacobi iteration method, Gauss- seidel iteration method, matrix inversion method, matrix factorization method etc. When the coefficient matrix and the right vector are in the interval form then also we may find the solution by Gauss-Elimination type method (interval from).In this context different authors develop different methods. Kolev [1] developed a method for outer interval solution of linear parametric systems. Skalna [2] introduced methods for solving systems of linear equations of structure mechanics with interval parameters. Popova [3] also introduced a method on the solution of parametrised linear systems. Walter Kraemer [4] investigated computing and visualizing solution sets of interval linear systems. Suparna Das, Diptiranjana Behera and S. Chakraverty recently find out a numerical solution of interval and fuzzy system of linear equation [6],[7]. In the present paper we have proposed a new method for solving interval system of linear equations which is undoubtedly iterative method for solving this type of system of equation and always gives strong solution.

## 2. RESEARCH METHOD

The interval form of the parameters may be written as

$$[\underline{x}, \bar{x}] = \{x : x \in \mathfrak{R}, \underline{x} \leq x \leq \bar{x}\}$$

where  $\underline{x}$  is the left value and  $\bar{x}$  is the right value of the interval respectively. We define  $m = \frac{x + \bar{x}}{2}$  is the centre and  $w = \bar{x} - \underline{x}$  is the width of the interval  $[\underline{x}, \bar{x}]$ .

Let  $[\underline{x}, \bar{x}]$  and  $[\underline{y}, \bar{y}]$  be two elements then the following arithmetic are well known [5]

- (i)  $[\underline{x}, \bar{x}] + [\underline{y}, \bar{y}] = [\underline{x} + \underline{y}, \bar{x} + \bar{y}]$
- (ii)  $[\underline{x}, \bar{x}] - [\underline{y}, \bar{y}] = [\underline{x} - \bar{y}, \bar{x} - \underline{y}]$
- (iii)  $[\underline{x}, \bar{x}] \times [\underline{y}, \bar{y}] = [\min\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}, \max\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}]$
- (iv)  $[\underline{x}, \bar{x}] \div [\underline{y}, \bar{y}] = [\min\{\underline{x} \div \underline{y}, \underline{x} \div \bar{y}, \bar{x} \div \underline{y}, \bar{x} \div \bar{y}\}, \max\{\underline{x} \div \underline{y}, \underline{x} \div \bar{y}, \bar{x} \div \underline{y}, \bar{x} \div \bar{y}\}]$

Let us consider the system of linear equation

$$\mathbf{Ax} = \mathbf{S}$$

Where  $[A] = [\bar{A}, \underline{A}]$  is an interval matrix of order  $n$  and  $[S] = [\bar{S}, \underline{S}]$  is an interval vector of  $n$  components. This linear system is called interval linear system

We can also write a linear system of interval equations explicitly as given below.

$$\begin{bmatrix} [a_{11}, b_{11}] & [a_{12}, b_{12}] & \cdots & [a_{1n}, b_{1n}] \\ [a_{21}, b_{21}] & [a_{22}, b_{22}] & \cdots & [a_{2n}, b_{2n}] \\ \vdots & \vdots & \vdots & \vdots \\ [a_{n1}, b_{n1}] & [a_{n2}, b_{n2}] & \cdots & [a_{nn}, b_{nn}] \end{bmatrix} \begin{bmatrix} [x_1, y_1] \\ [x_2, y_2] \\ \vdots \\ [x_n, y_n] \end{bmatrix} = \begin{bmatrix} [c_1, d_1] \\ [c_2, d_2] \\ \vdots \\ [c_n, d_n] \end{bmatrix} \tag{1}$$

where the members in an interval are both either positive or negative.

The system of equations in equation (1) can be solved by direct elimination method which we generally do for the crisp data. Now interval values are operated through the interval arithmetic rules. Because of the more computation involved in this procedure it is a difficult task to perform and the margin of uncertainty increases drastically. To overcome the above difficulties (to a certain extent) we now propose a new method.

Equation (1) can be solved by iterative scheme like Gauss-Seidel numerical method. The new form of representation of Equation (1) is as follows

$$\begin{aligned} [x_1, y_1]^{(k+1)} &= \frac{[c_1, d_1] - [a_{12}, b_{12}][x_2, y_2]^{(k)} - [a_{13}, b_{13}][x_3, y_3]^{(k)} - \cdots - [a_{1n}, b_{1n}][x_n, y_n]^{(k)}}{[a_{11}, b_{11}]} \\ [x_2, y_2]^{(k+1)} &= \frac{[c_2, d_2] - [a_{21}, b_{21}][x_1, y_1]^{(k)} - [a_{23}, b_{23}][x_3, y_3]^{(k)} - \cdots - [a_{2n}, b_{2n}][x_n, y_n]^{(k)}}{[a_{22}, b_{22}]} \\ &\dots \\ &\dots \\ &\dots \\ [x_n, y_n]^{(k+1)} &= \frac{[c_n, d_n] - [a_{n1}, b_{n1}][x_1, y_1]^{(k)} - [a_{n2}, b_{n2}][x_2, y_2]^{(k)} - \cdots - [a_{n(n-1)}, b_{n(n-1)}][x_{n-1}, y_{n-1}]^{(k)}}{[a_{nn}, b_{nn}]} \end{aligned} \tag{2}$$

The above procedure will be more efficient if we replace the vector  $[x, y]^{(k)}$  in the right side of Equation (2) element by element as in Gauss-Jordan type method. So Equation (2) may then be written as

$$\begin{aligned} [x_1, y_1]^{(k+1)} &= \frac{[c_1, d_1] - [a_{12}, b_{12}][x_2, y_2]^{(k)} - [a_{13}, b_{13}][x_3, y_3]^{(k)} - \cdots - [a_{1n}, b_{1n}][x_n, y_n]^{(k)}}{[a_{11}, b_{11}]} \\ [x_2, y_2]^{(k+1)} &= \frac{[c_2, d_2] - [a_{21}, b_{21}][x_1, y_1]^{(k+1)} - [a_{23}, b_{23}][x_3, y_3]^{(k)} - \cdots - [a_{2n}, b_{2n}][x_n, y_n]^{(k)}}{[a_{22}, b_{22}]} \\ &\dots \\ &\dots \\ &\dots \\ [x_n, y_n]^{(k+1)} &= \frac{[c_n, d_n] - [a_{n1}, b_{n1}][x_1, y_1]^{(k+1)} - [a_{n2}, b_{n2}][x_2, y_2]^{(k+1)} - \cdots - [a_{n(n-1)}, b_{n(n-1)}][x_{n-1}, y_{n-1}]^{(k)}}{[a_{nn}, b_{nn}]} \end{aligned} \tag{3}$$



Table 2. Comparison /summary of methods/results

Temperatures (in degree Celsius)	Traditional Finite element method (crisp)(for two element discretization )	Traditional Finite element method (crisp) (for four element discretization) (series1)	Interval analysis	Interval analysis (series 2)	Interval analysis (series 3)	Interval analysis (series 4)
-	-	-	Usual method(for two element discretization)	Usual method(for two element discretization)	Proposed Method	Method [8]
$T_1$	95.1414	95.2980	95.1756	95.695	95.205	95.1431
$T_2$	-	90.3024	-	90.625	90.235	90.1423
$T_3$	85.1414	85.3068	85.16045	85.44	85.21	85.14155
$T_4$	-	82.7380	-	82.73	82.865	82.5708
$T_5$	80	80	80	80	80	80

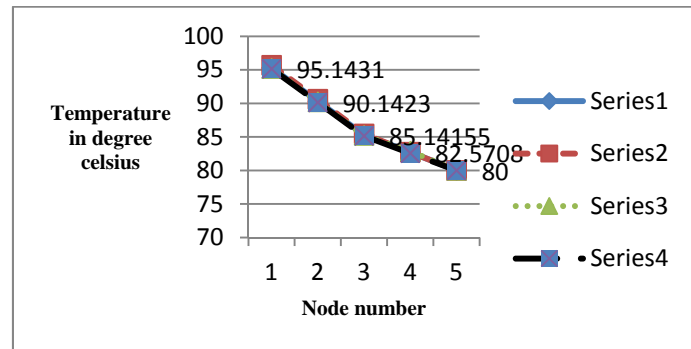


Figure.1. Compare different methods (for four element discretization)

Here we found that the marginal error caused due to the uncertain parameter used in the said real world problem becomes more for the usual interval finite element method which is shown in the Table-1. Whereas using proposed method we get a comparatively better result. The possibility of the solution set is decreased, which gives a better picture to predict the temperatures at the nodal point of the domain .Again if we consider the centre value of the solution set and comparing the two results of Table-1and Figure1, proposed method gives a better approximation compare to the results of the others.

#### 4. CONCLUSION

In this paper we proposed a method for solving a interval system of linear equation which is relatively new approach for solving linear equation besides other approaches .By this method we can easily find out a strong solution with better result.

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