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TM-Polarization One-Dimensional Photonic Crystal Design

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ABSTRACT

A theoretical investigation of one dimensional planar photonic crystal is carried out. These photonic crystal consist of a dielectric layer structures with refractive index n_1=1.45 and n_2= 3.45. This work presents a systematic investigation of the reflection, forbidden bands and density of state of p-polarization. In optical sciences, the refractive index of an optical medium is a most fundamental quantity. The refractive index determines the refraction and reflection occurring at the boundary between two media. The propagation angle in one medium is taken with respect to normal inside the first medium varies between 0 and π/2. The program is written in MATLAB to simulate and analysis dispersions of electric magnetic waves in one dimension photonic crystal.

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1. INTRODUCTION

Photonic crystals (PCs) are structures with periodically modulated dielectric constants whose distribution follows a periodicity of the order of a fraction of the optical wavelength. Since the first pioneering work in this field, many new interesting ideas have been developed dealing with one dimensional (1D) [1]. The simplest case of a photonic crystal is one dimensional photonic crystal. It is a structure built of alternating layers of linear, uniform and isotropic materials of two different refractive indices n_1 and n_2 [2]. Its section in xz plane is shown on Figure. 1. The structure has rotational symmetry around z axis, all layers extend towards infinity in x and y directions (direction y is perpendicular to section shown on Figure 1) and the photonic crystal built of finite number of layers. A primitive cell in this case is a pair of subsequent layers with refractive indices n_1 and n_2. The number of primitive cells right from $z = 0$ plane= N_R and to the left=N_L, where directions right and left are in accordance with those on Figure 1.

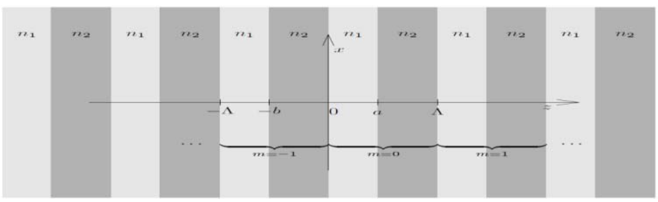

Figure. 1 Structure of one-dimensional photonic crystal (section) [2].

The primitive cell index m as depicted on the Figure: layers with $z \in [0, \Lambda]$ constitute primitive cell with m, and the next (along with the z axis) pair has $m = 1$, the first one has $m = N_R - 1$, the first primitive cell left from $z = 0$ plane has $m = -1$ and the index of the last one is $m = -N_L$. The crystal is surrounded by a material with refractive index n_1 (for simplicity) [2].

The idea of the described method is presented on Figure 2. It aims to determine effective reflection from a part of photonic crystal on a layer's boundary and replace it with a mirror of the same reflection. In general case of photonic crystal, precise description of density of states (DOS) without approximations requires difficult and time consuming calculations.

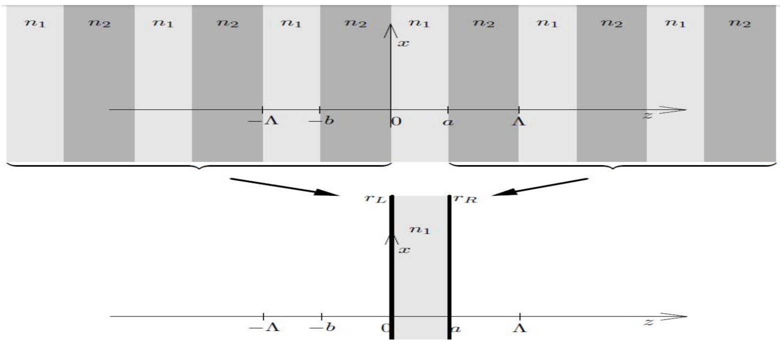

Figure 2 A layer of one dimensional photonic crystal as a resonator [2].

However, for the simplest case of one-dimensional photonic crystal it is possible to follow the derivation given below in the further part of the paper. In our model we assume that in each of layers of the photonic crystal Maxwell equations have solutions in form of plane waves [2]:

$$\vec{E} = \vec{E}_0 e^{i\vec{k}\vec{r} - i\omega t} \tag{1}$$

with wave vector $\vec{k}$ bound with angular frequency ω by dispersion relation [2]: $\vec{k}^2 = \frac{n^2\omega^2}{c^2}$.
where c is speed of light in vacuum and n is refractive index of the material of the layer. We can relate fields in different layers using continuity conditions. In every n_1 layer, the electric field as a superposition of coupled plane waves [2]:

$$E_{m,1,\in} = a_{m,\in} e^{ik_x x + i\beta_j z - i\omega t} + b_{m,\in} e^{ik_x x - i\beta_j z - i\omega t} \quad , z \in [m\Lambda, m\Lambda + a] \tag{2}$$

where $\in$ denotes polarization TM (electric field in plane of incidence) and [2],

$$\beta_j = \sqrt{\frac{n_j^2\omega^2}{c^2} - k_x^2} \quad , j = 1,2 \tag{3}$$

and similarly in every n_2 layer. Continuity conditions lead to equation relating amplitudes in consecutive primitive cells [2]: $\begin{bmatrix} a_{m+1,\in} \\ b_{m+1,\in} \end{bmatrix} = M_{m,\in} \begin{bmatrix} a_{m,\in} \\ b_{m,\in} \end{bmatrix}$, where $M_{m,\in}$ is translation matrix of $m-th$ primitive cell. It is an obvious conclusion, that if amplitudes of plane waves outside the photonic crystal are indexed with $m = N_R$ for right and $m = -N_L - 1$ for left, then: $M_{m,\in} = M_{m',\in} \equiv M_{\in}$, where, $M_{\in} = \begin{bmatrix} A_{\in} & B_{\in} \\ C_{\in} & D_{\in} \end{bmatrix}$ and [2]:

$$A_{TM} = e^{i\beta_1 a}(\cos(\beta_2 b) + \frac{i}{2}(\frac{n_2^2\beta_1}{n_1^2\beta_2} + \frac{n_1^2\beta_2}{n_2^2\beta_1})\sin(\beta_2 b)) \qquad B_{TM} = \frac{i}{2}e^{i\beta_1 a}(\frac{n_1^2\beta_2}{n_2^2\beta_1} - \frac{n_2^2\beta_1}{n_1^2\beta_2})\sin(\beta_2 b)$$

$$C_{TM} = \frac{i}{2}e^{i\beta_1 a}(\frac{n_2^2\beta_1}{n_1^2\beta_2} - \frac{n_1^2\beta_2}{n_2^2\beta_1})\sin(\beta_2 b) \qquad D_{TM} = e^{-i\beta_1 a}(\cos(\beta_2 b) + \frac{i}{2}(\frac{n_2^2\beta_1}{n_1^2\beta_2} + \frac{n_1^2\beta_2}{n_2^2\beta_1})\sin(\beta_2 b)) \tag{4}$$

As a result, is simplified to [2]: $\begin{bmatrix} a_{N,\in} \\ b_{N,\in} \end{bmatrix} = M_{\in}^{N} \begin{bmatrix} a_{0,\in} \\ b_{0,\in} \end{bmatrix}$, where, $a_{0,\in}$ and $b_{0,\in}$ = amplitudes.

Because the bexcitation in the photonic crystal (or at least on the other side of it) and there is no reflection in the infinity. Because $a_{0,\in}$ is the amplitude of the wave incident on layer boundary in $z = a, b_{0,\in}$ can be treated as reflected wave. Therefore, we can calculate reflection coefficient of the whole part of photonic crystal reflecting $a_{0,\in}$ wave, it will be [2]:

$$r_{\in} = \frac{b_{0,\in} e^{-i\beta_1 a}}{a_{0,\in} e^{i\beta_1 a}} \tag{5}$$

2. DENSITY OF STATE IN 1D-PHOTONIC CRYSTAL

Photonic crystals have been the subject of intense investigation due to their ability to control the properties of photons [3]. When a collimated light ray of wavelength λ in a homogeneous medium (e.g. *air)* reaches the surface of the 1D photonic crystal slab at an incidence angle θ_{inc}. For simplicity we consider the case where the direction of wave propagation is restricted in the xy plane (Figure 3). After entry into the photonic crystal, the light ray propagates at an angle of refraction θ_{pc}. To compute the relationship between θ_{inc} and θ_{pc} for a given wavelength, we match the frequency and tangential component of the wave vector for the incident and refracted wave across the interface using the following simple procedure. We specify the angular frequency $\omega = 2\pi f$ (f is the frequency of light) and the incidence angle θ_{inc} in the homogeneous medium. Using the relationships $\varepsilon_r \omega^2 = k_x^2 + k_y^2$ and $\tan(\theta_{inc}) = (k_y / k_x)$ we can find the wave vector in the incident medium. Here ε_r is the relative permittivity of the incident medium, and k_x and k_y are the components of the wave vector perpendicular and parallel, respectively, to the interface between the homogeneous medium and the photonic crystal. (We assume $k_z = 0$ for simplicity.) We can compute the 1D photonic crystal dispersion relationship, or photonic band structure, $\omega(k)$, using the transfer matrix method, a standard technique commonly found in the literature. Here the angular frequency ω and the parallel component of the wave vector k_y, are the same as those in the incident homogeneous medium. The transfer matrix technique allows us to find the perpendicular component of the wave vector k_x in the photonic crystal. From $\omega(k)$ we can then compute the photonic crystal group velocity using $v_g = \nabla_k \omega(k) = (\partial\omega(k)/\partial k_x, \partial\omega(k)/\partial k_z, \partial\omega(k)/\partial k_z)$. From the components of the group velocity we can determine the angle of refraction using $\theta_{pc} = \tan^{-1}(v_{g-y} / v_{g-x})$ [4].

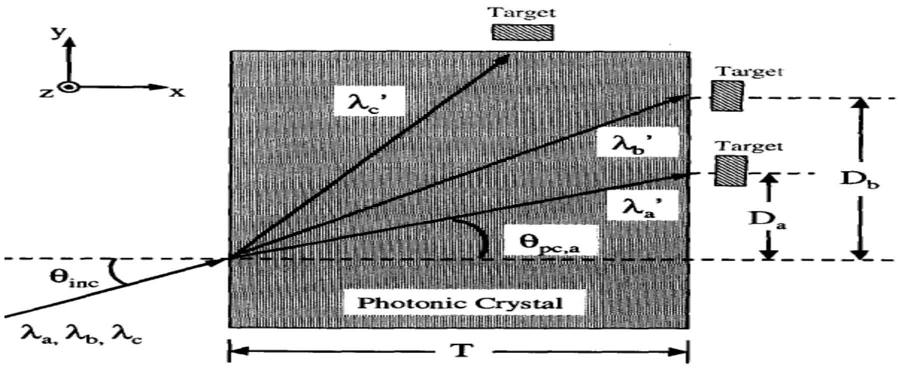

Figure 3 Illustration of how a 1D photonic crystal might be used to disperse light of different wavelengths [4]

In recent years, high quality factor, small-mode volume cavities based on photonic crystals have attracted significant attention, because of their ability to modify the density of optical states strongly. An increase in DOS of the lasing mode causes significant enhancement of the spontaneous emission rate [5]. The concepts of density of state have proven essential in the study of electromagnetic wave propagation through periodic structures. The DOS is usually defined as [6]:

$$N(\omega) = \sum_m \frac{1}{A_{BZ}} \int_{BZ} \delta(\omega - \omega_m(k)) d^2k \tag{6}$$

where the integral is taken over the $m-th$ band and A_{BZ} is the area of the Brillouin zone (BZ). Eq. (6) can also be written as [6]:

$$N(\omega) = \sum_m \frac{1}{A_{BZ}} \int_{EFS_m} \left\| \frac{dk}{d\omega} \right\| ds \tag{7}$$

where the integral is taken along the $m-th$ equifrequency surfaces (EFS) at frequency ω and $v_g^{-1} = dk/d\omega$ is the inverse group velocity. The DOS was first used in understanding the modification of spontaneous emission in photonic crystals. The DOS plays an important role in light trapping for solar cells and in mode confinement in photonic crystal structures. The definition in equation (6) suggests the typical method by which the DOS is computed: using the full band structure and binning by frequency to approximate the integral. The frequency binning method can be improved if the group velocities are also available [2], [6]. Photonic crystals are strongly wavelength sensitive, high index contrast dielectric materials. This sensitivity originates from the dispersive properties of the photonic crystal structure as a result of the wavelength scale refractive index modulation with certain crystal symmetry [7]. The behavior of a photon with a certain frequency will depend on the propagation direction within the photonic crystals. The modulation of the refractive index will cause that certain energies and directions are forbidden for photons. A region of energies where the photonic crystal does not allow photons to propagate regardless of their direction and polarization is called a complete photonic band gap (cPBG) [8].

3. RESULTS AND ANALYSIS

We proceed with the calculation of fields for TE modes. MATLAB is a great and easy tool to use to simulate optical electronics. All the results below are got after following these steps:

1. Calculate the reflectance function.
2. Implementation of dispersion relation of electromagnetic waves in 1D photonic crystal.
3. Found the normalized frequency.
4. Found the ray angle with respect to normal inside 1st medium varies between 0-$\pi/2$ and inside the 2nd medium using Snell law ($n_1 = 1.45$ and $n_2 = 3.45$). Then transform incidence angle in degrees.
5. Select points which belong to the forbidden bands.
6. In order to compute the DOS we found the inverse group velocity at each sample point and then the DOS can be directly obtained using equation (7).

Figure 4 is about the reflectance function versus wavelength with average mean values=0.8055, median=0.8584 and standard deviation (STD)=0.2673. The magnitude of reflectance=1 within the range of infrared. Figure 5 shows forbidden bands p-polarization, it explain the relationship between incidence angles versus normalized frequency. The incidence angle is an important parameter which effects the width of band gaps. The mean= 0.572, median= 0.6057, mode= 0.3084 and STD= 0.2586. The thicknesses and the index contrast of the photonic crystal determinate many of its optical properties. Playing on these two parameters, we can obtain frequency ranges for which light propagation is forbidden in the material and others ranges for which light can propagate. These frequency ranges are also scale dependent. Reducing the size of the elementary cell of the periodic lattice shifts the whole frequency range to higher values. Figure 6 represents the relationship between the normalized frequencies versus density of state for M polarization.

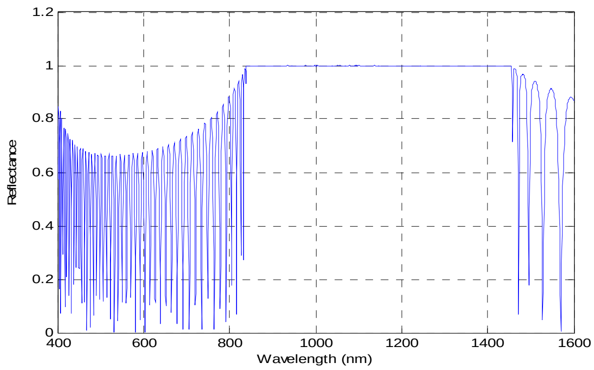

Figure 4 Reflectance versus the wavelength. The reflectance is extremely high over the range $\approx 825 - 1460\,nm$.

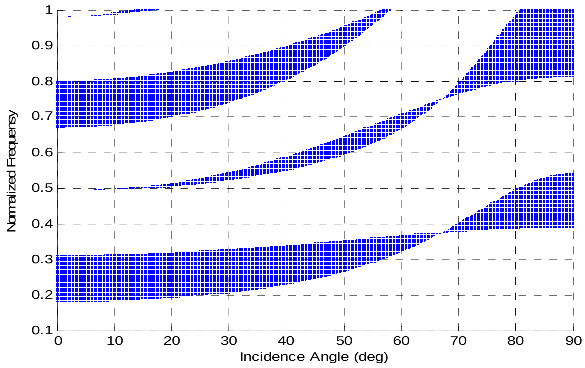

Figure 5 Incidence angle versus normalized frequency. It shows the forbidden bands for p-polarization. Not that, the larger the difference between the two indices the wider the band gaps become.

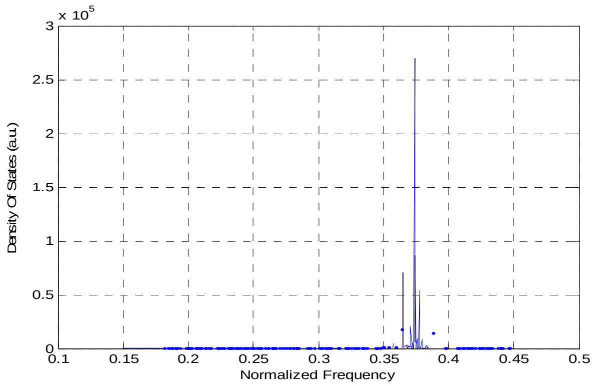

Figure 6 Density of state versus normalized frequency. Minimum, maximum DOS are between =0.0137 to infinity and the mediam=5.81.

4. CONCLUSION

We can construct very compact optical devices with desired optical properties such as lasers, light emitting diodes, filters, waveguides, holey fibers, and photonic integrated circuits by designing specific photonic crystal structures and introducing some defects in a PC,.

The reflection coefficient for a plane wave incident upon a periodic dielectric structure was analyzed. The research presented in this paper focuses on planar one dimensional photonic structures consists of alternating layers of material with different dielectric constants ($n_1 = 1.45$ and $n_2 = 3.45$). It has been shown that the larger the difference between the two indices the wider the band gaps become. Also, as the width of the air layers become smaller in comparison to the width of the dielectric layers, the width of band gaps would decrease. The angle of incidence of the light wave is also another factor which effects the width of band gaps. DOS is an important factor in light trapping in photonic crystal structures.

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